華中科技大學
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# 《电力系统分析》（I）

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# 第三章 同步发电机的基本方程式

## 本章主要内容

根据理想同步电机内部的各电磁量的关系，建立比较完整精确的同步电机数学模型，为电力系统暂态分析准备必要的知识

# 第三章 同步发电机的基本方程式

3-1 基本前提

3-2 同步电机原始方程式

3-3 dq0坐标系的同步电机方程

3-6 同步电机的对称稳态运行

# 第三章 同步发电机的基本方程式

1. 同步电机原始方程式：各绕组及相互间电感系数时变；电压方程式时变系数微分方程，不便于求解；

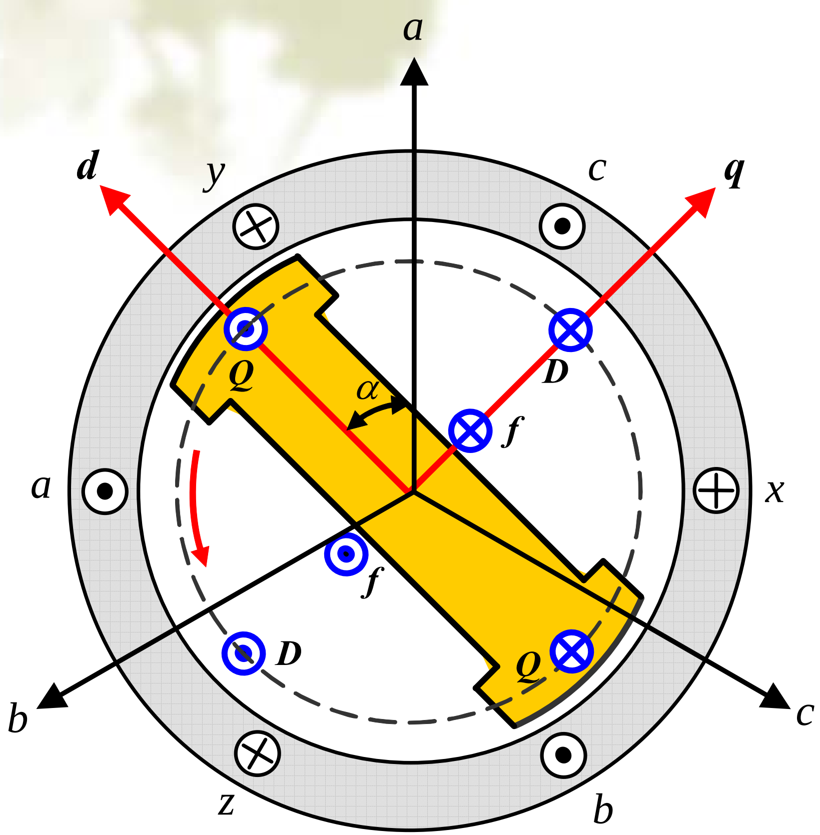
2. 同步电机Park方程式：定子侧各绕组方程式从abc坐标系转换到dq0坐标系，得到常系数微分方程；—Park变换

3. 同步电机对称稳态运行：根据同步电机Park方程式，得到用相量表示的稳态电势方程式，等值电路，相量图；空载电势Eq和等值隐极机电势EQ的定义；

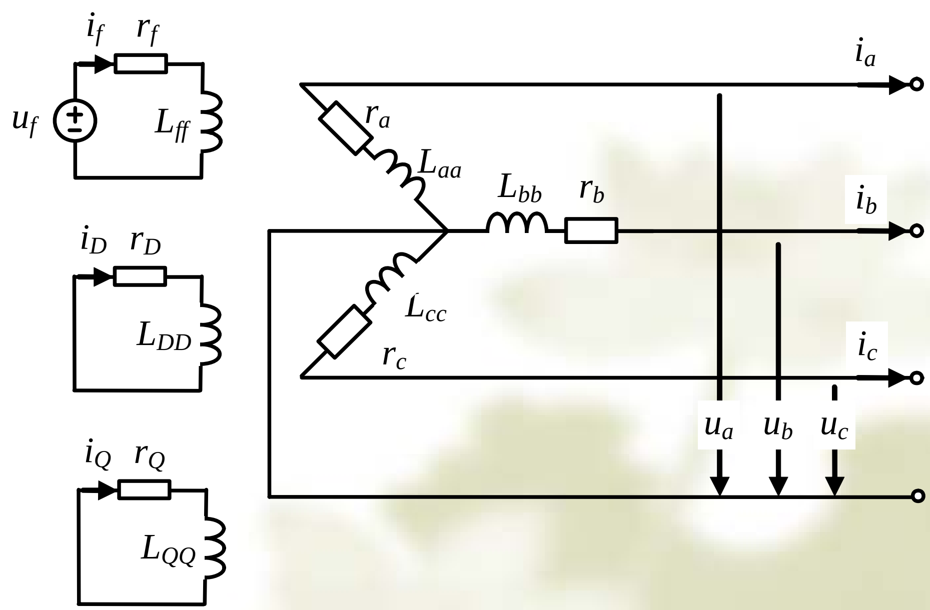
Ex3-2：已知机端电压电流，计算同步电机空载电势及其相角；

# 3-1 基本前提

## 同步电机基本回路图（理想同步电机假设、假定正方向）




绕组轴线正向示意图



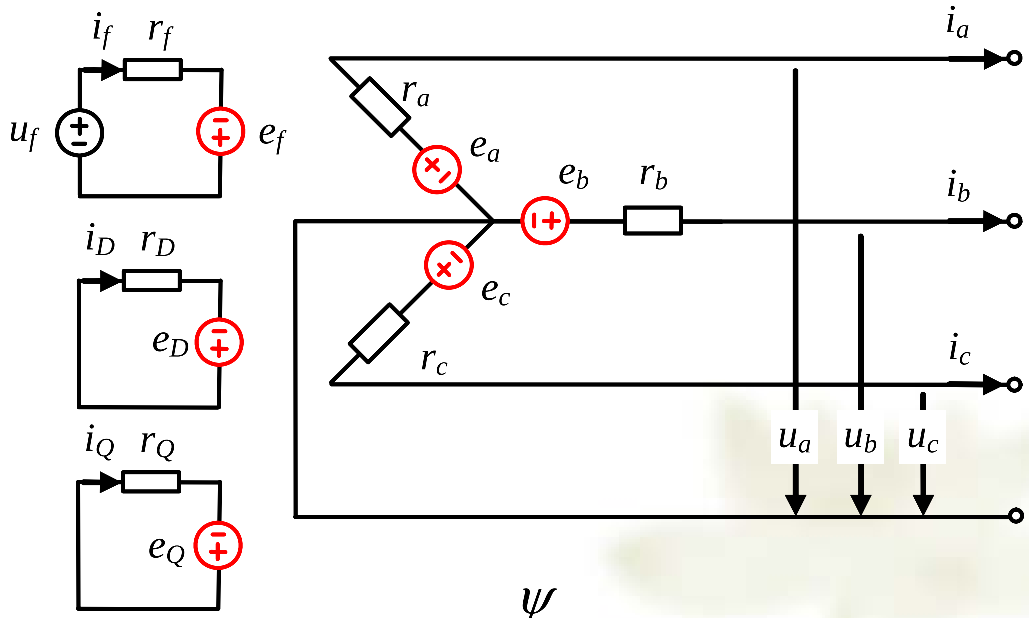

基本回路图（未标出互感）

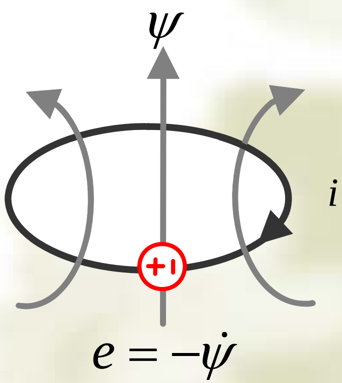
# 3-2 同步电机的原始方程式

## 1. 电势方程

$$
\begin{cases}
u_a = e_a - r_a i_a = -\dot{\psi}_a - r_a i_a \\
u_b = e_b - r_b i_b = -\dot{\psi}_b - r_b i_b \\
u_c = e_c - r_c i_c = -\dot{\psi}_c - r_c i_c
\end{cases}
$$

$$
\begin{cases}
u_f = -e_f + r_f i_f = \dot{\psi}_f + r_f i_f \\
0 = -e_D + r_D i_D = \dot{\psi}_D + r_D i_D \\
0 = -e_Q + r_Q i_Q = \dot{\psi}_Q + r_Q i_Q
\end{cases}
$$



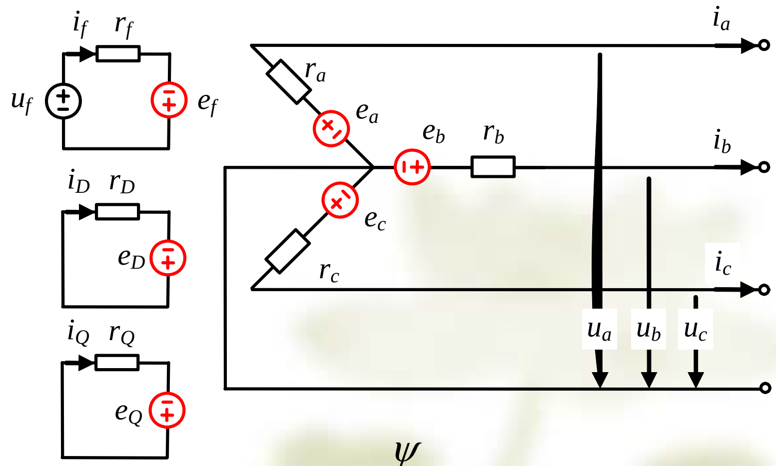



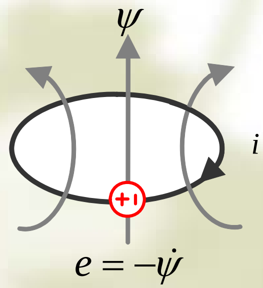
# 3-2 同步电机的原始方程式

## 1. 电势方程

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ \hdashline -u_f \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} \dot{\psi}_a \\ \dot{\psi}_b \\ \dot{\psi}_c \\ \hdashline \dot{\psi}_f \\ \dot{\psi}_D \\ \dot{\psi}_Q \end{bmatrix} - \left[\begin{array}{ccc:ccc} r & 0 & 0 & & & \\ 0 & r & 0 & & \mathbf{0} & \\ 0 & 0 & r & & & \\ \hdashline & & & r_f & 0 & 0 \\ & \mathbf{0} & & 0 & r_D & 0 \\ & & & 0 & 0 & r_Q \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hdashline i_f \\ i_D \\ i_Q \end{bmatrix}$$

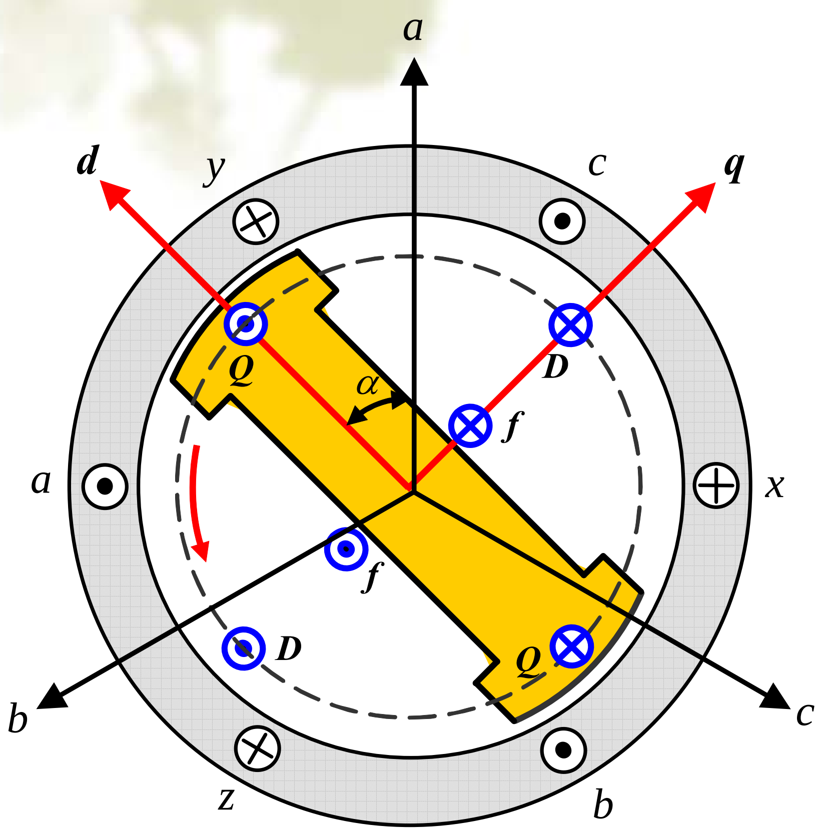
$$\begin{bmatrix} \mathbf{u}_{abc} \\ \mathbf{u}_{fDQ} \end{bmatrix} = -\begin{bmatrix} \dot{\boldsymbol{\psi}}_{abc} \\ \dot{\boldsymbol{\psi}}_{fDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{r}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{r}_R \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{fDQ} \end{bmatrix}$$



# 3-2 同步电机的原始方程式

## 2. 磁链方程




绕组轴线正向示意图

$$
\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \hdashline \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix}
=
\left[\begin{array}{ccc:ccc}
L_{aa} & L_{ab} & L_{ac} & L_{af} & L_{aD} & L_{aQ} \\
L_{ba} & L_{bb} & L_{bc} & L_{bf} & L_{bD} & L_{bQ} \\
L_{ca} & L_{cb} & L_{cc} & L_{cf} & L_{cD} & L_{cQ} \\
\hdashline
L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fD} & L_{fQ} \\
L_{Da} & L_{Db} & L_{Dc} & L_{Df} & L_{DD} & L_{DQ} \\
L_{Qa} & L_{Qb} & L_{Qc} & L_{Qf} & L_{QD} & L_{QQ}
\end{array}\right]
\begin{bmatrix} i_a \\ i_b \\ i_c \\ \hdashline i_f \\ i_D \\ i_Q \end{bmatrix}
$$

$$
\begin{bmatrix} \boldsymbol{\Psi}_{abc} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix}
=
\begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix}
\begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{fDQ} \end{bmatrix}
$$

# 3-2 同步电机的原始方程式

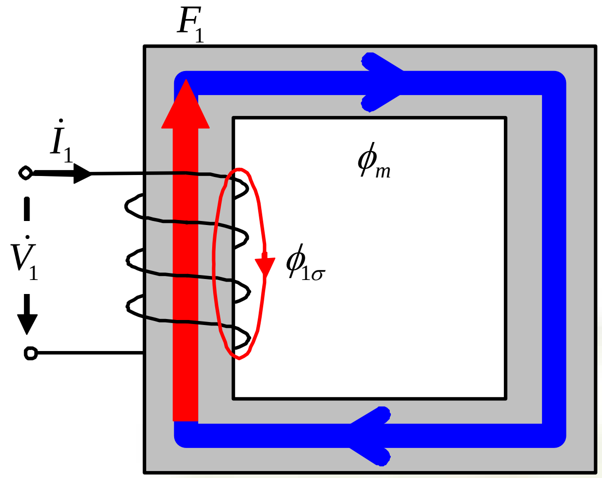
## 3. 电感系数

| | | | |
|---|---|---|---|
| 定子绕组自感系数 | $L_{aa}$ | $L_{bb}$ | $L_{cc}$ |
| 定子绕组间互感系数 | $L_{ab}=L_{ba}$ | $L_{ab}=L_{ba}$ | $L_{ab}=L_{ba}$ |
| 转子绕组自感系数 | $L_{ff}=L_{f}$ | $L_{DD}=L_{D}$ | $L_{QQ}=L_{Q}$ |
| 转子绕组间互感系数 | $L_{fD}=L_{Df}$ | $L_{fQ}=L_{Qf}$ | $L_{DQ}=L_{QD}$ |
| 定转子绕组间互感系数 | $L_{af}=L_{fa}$ | $L_{bf}=L_{fb}$ | $L_{cf}=L_{fc}$ |
| | $L_{aD}=L_{Da}$ | $L_{bD}=L_{Db}$ | $L_{cD}=L_{Dc}$ |
| | $L_{aQ}=L_{Qa}$ | $L_{bQ}=L_{Qb}$ | $L_{cQ}=L_{Qc}$ |

# Review：磁路欧姆定律

$$F_1 = w_1 i_1$$

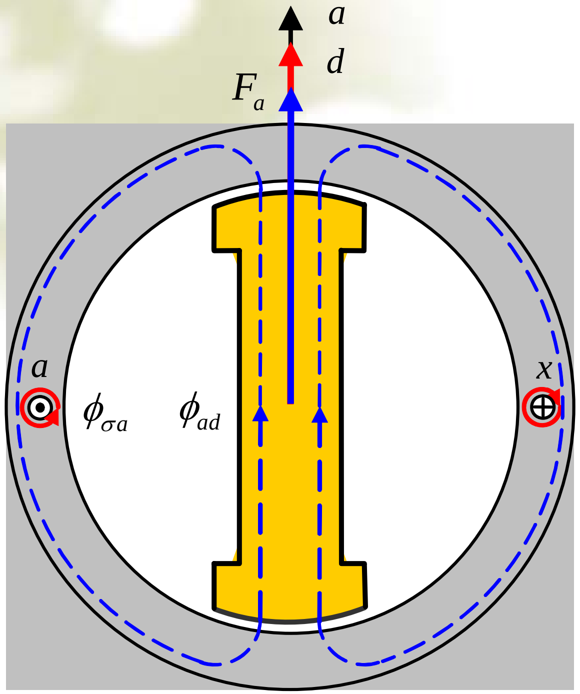
$$\phi_{1\sigma} = F_1 \lambda_{s\sigma},\ \phi_m = F_1 \lambda_m$$



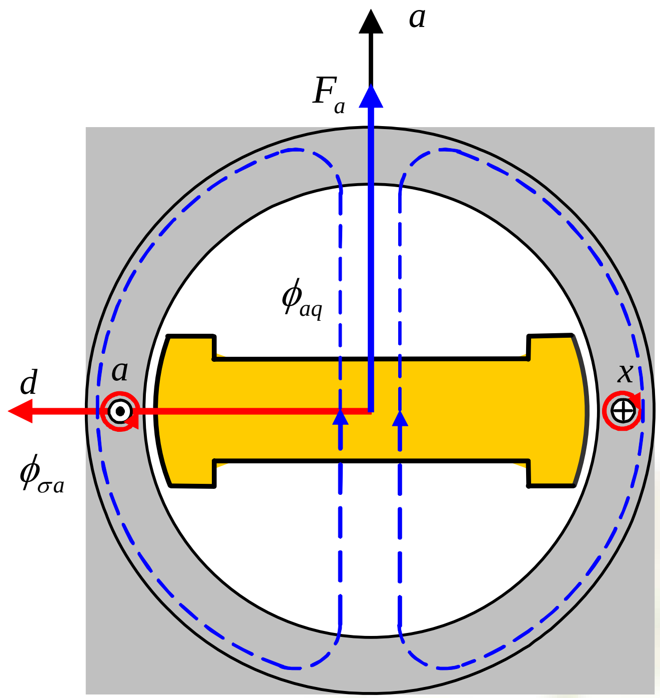

$$\psi_1 = w_1 \left(\phi_{1\sigma} + \phi_m\right) = w_1^2 \left(\lambda_{s\sigma} + \lambda_m\right) i_1$$

$$L_1 = \psi_1 / i_1 = w_1^2 \left(\lambda_{s\sigma} + \lambda_m\right)$$

# (1)定子绕组自感系数——以a相为例




$\alpha = 0^\circ$




$\alpha = 90^\circ$

$$F_a = w_a i_a,\ \phi_{\sigma a} = F_a \lambda_{s\sigma},\ \phi_{ad} = F_a \lambda_{ad}$$

$$\psi_a = w_a\left(\phi_{\sigma a} + \phi_{ad}\right) = w_a^2\left(\lambda_{s\sigma} + \lambda_{ad}\right) i_a$$

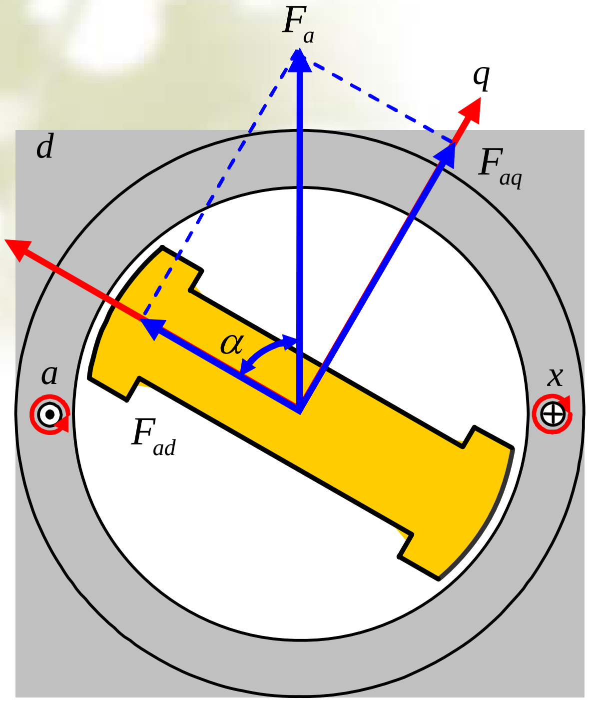
$$L_{aa} = \psi_a / i_a = w_a^2\left(\lambda_{s\sigma} + \lambda_{ad}\right)$$

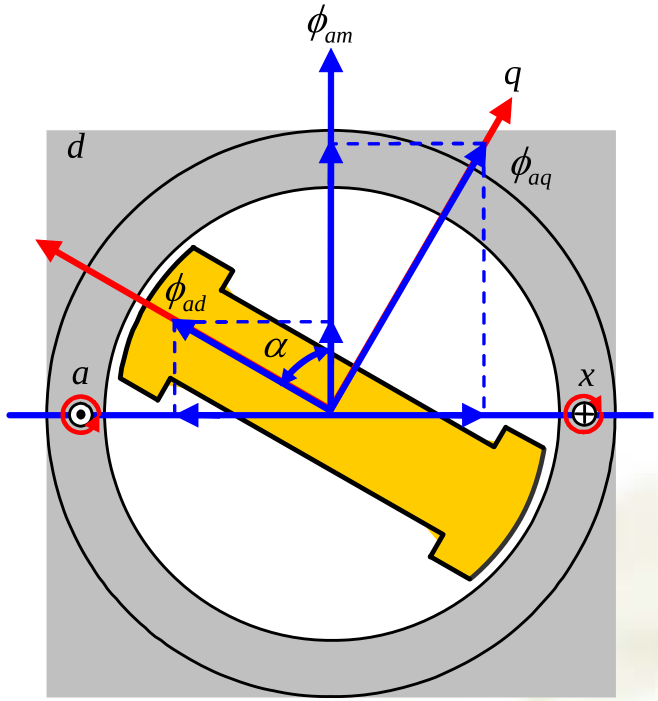
$$F_a = w_a i_a,\ \phi_{\sigma a} = F_a \lambda_{s\sigma},\ \phi_{aq} = F_a \lambda_{aq}$$

$$\psi_a = w_a\left(\phi_{\sigma a} + \phi_{aq}\right) = w_a^2\left(\lambda_{s\sigma} + \lambda_{aq}\right) i_a$$

$$L_{aa} = \psi_a / i_a = w_a^2\left(\lambda_{s\sigma} + \lambda_{aq}\right)$$

# (1)定子绕组自感系数——以a相为例



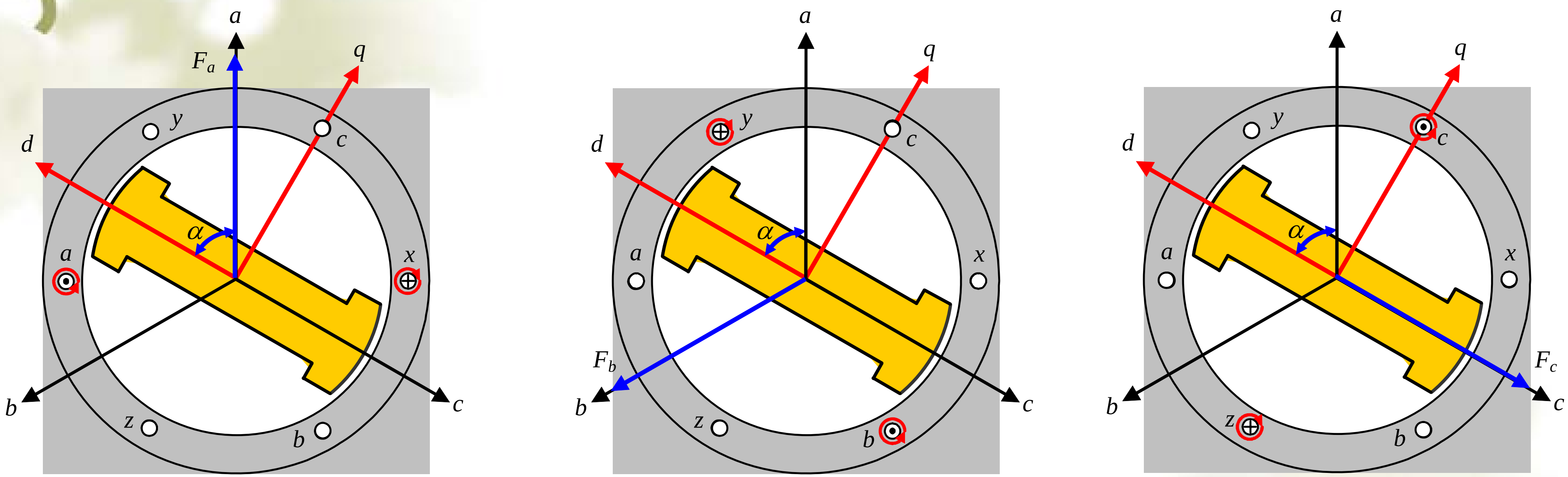

$$F_a = w_a i_a \Leftrightarrow F_{ad} = F_a \cos\alpha,\ F_{aq} = F_a \sin\alpha$$

$$\phi_{\sigma a} = F_a \lambda_{s\sigma},\ \phi_{ad} = F_{ad}\lambda_{ad},\ \phi_{aq} = F_{aq}\lambda_{aq}$$

$$\psi_a = w_a\left(\phi_{\sigma a} + \phi_{ad}\cos\alpha + \phi_{aq}\sin\alpha\right) = w_a^2\left(\lambda_{s\sigma} + \lambda_{ad}\cos^2\alpha + \lambda_{aq}\sin^2\alpha\right)i_a$$

$$L_{aa} = \psi_a / i_a = w_a^2\left(\lambda_{s\sigma} + \lambda_{ad}\cos^2\alpha + \lambda_{aq}\sin^2\alpha\right) = l_0 + l_2\cos 2\alpha$$

# (1)定子绕组自感系数—abc三相

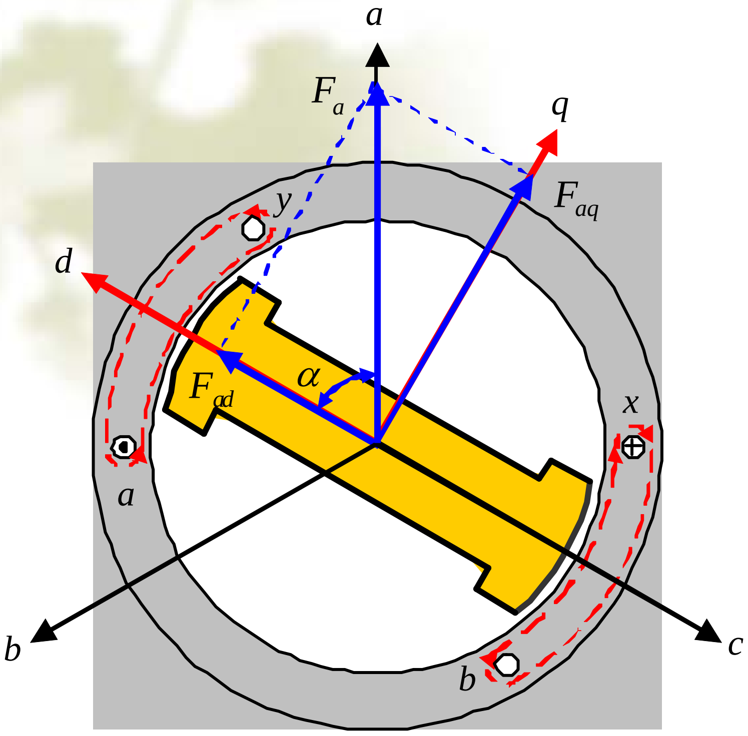


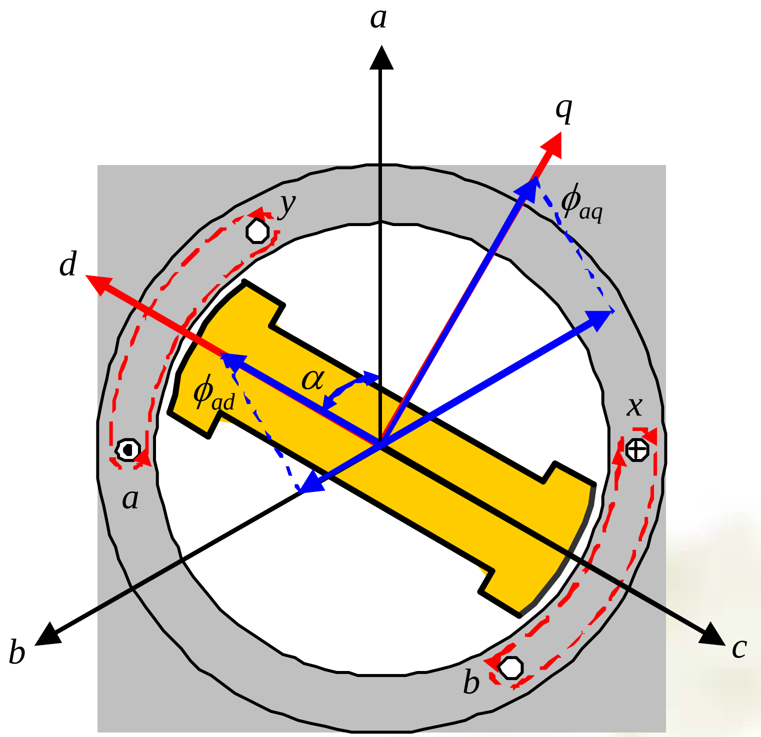

$$\begin{cases} L_{aa} = \psi_a / i_a = l_0 + l_2 \cos 2\alpha \\ L_{bb} = \psi_b / i_b = l_0 + l_2 \cos 2\left(\alpha - 120^\circ\right) \\ L_{cc} = \psi_c / i_c = l_0 + l_2 \cos 2\left(\alpha + 120^\circ\right) \end{cases}$$

$$\begin{cases} l_0 = w_a^2 \left( \lambda_{s\sigma} + \frac{1}{2}\left(\lambda_{ad} + \lambda_{aq}\right) \right) \\ l_2 = w_a^2 \times \frac{1}{2}\left(\lambda_{ad} - \lambda_{aq}\right) \end{cases}$$

**隐极同步电机：$l_2=0$，$L_{aa}=L_{bb}=L_{cc}=\text{const}$**

## (2)定子绕组间互感系数——以ab相为例




$$F_a = w_a i_a \Leftrightarrow F_{ad} = F_a \cos\alpha,\ F_{aq} = F_a \sin\alpha$$

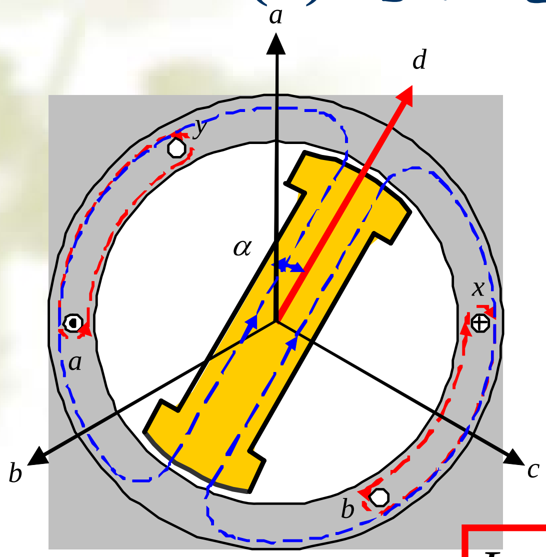
$$\phi_{ba\sigma} = -F_a \lambda_{m\sigma},\ \phi_{ad} = F_{ad}\lambda_{ad},\ \phi_{aq} = F_{aq}\lambda_{aq}$$

$$\psi_b = w_b\left(\phi_{ba\sigma} + \phi_{ad}\cos\left(\alpha - 120^\circ\right) + \phi_{aq}\sin\left(\alpha - 120^\circ\right)\right)$$

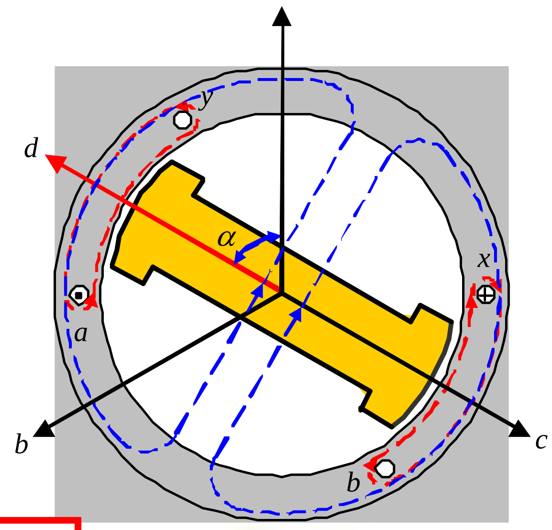
$$= w_a w_b\left(-\lambda_{m\sigma} + \lambda_{ad}\cos\alpha\cos\left(\alpha - 120^\circ\right) + \lambda_{aq}\sin\alpha\sin\left(\alpha - 120^\circ\right)\right) i_a$$

$$L_{ba} = L_{ab} = \psi_b / i_a = -\left[m_0 + m_2\cos 2\left(\alpha + 30^\circ\right)\right]$$

## (2)定子绕组间互感系数——以ab相为例

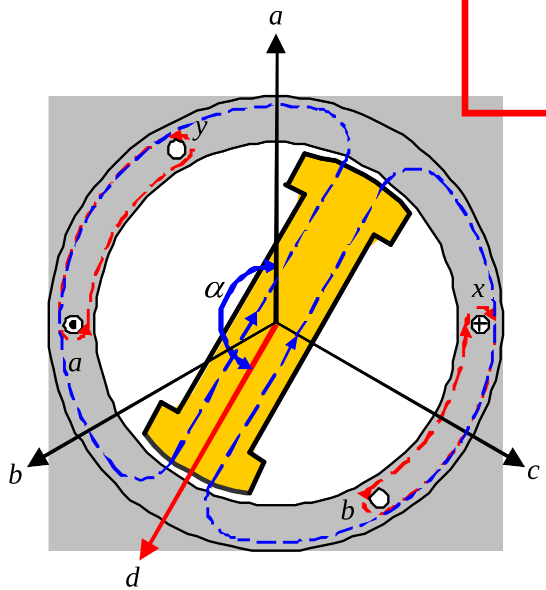



$\alpha = -30^{\circ}$

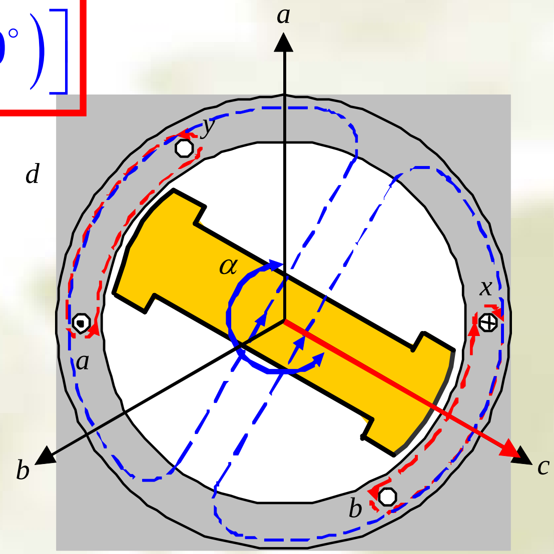



$\alpha = 60^{\circ}$

$$L_{ba} = L_{ab}$$
$$= -\left[ m_0 + m_2 \cos 2\left( \alpha + 30^{\circ} \right) \right]$$




$\alpha = 150^{\circ}$




$\alpha = 240^{\circ}$

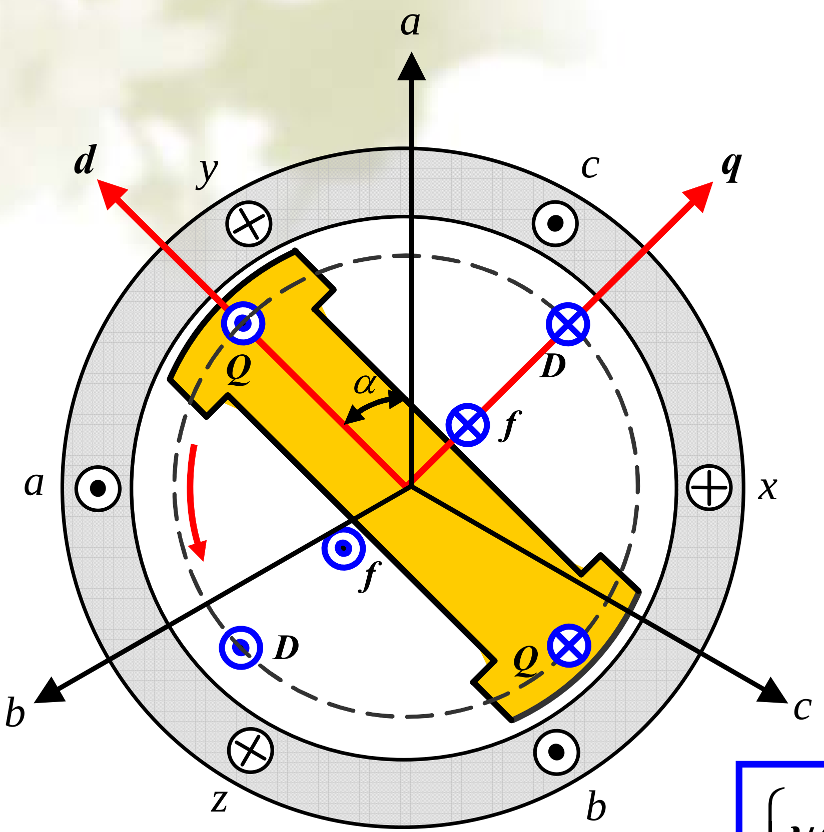
## (2)定子绕组间互感系数—abc三相

$$\begin{cases} L_{ba} = L_{ab} = -\left[ m_0 + m_2 \cos 2\left(\alpha + 30^\circ\right)\right] \\ L_{bc} = L_{cb} = -\left[ m_0 + m_2 \cos 2\left(\alpha - 90^\circ\right)\right] \\ L_{ca} = L_{ac} = -\left[ m_0 + m_2 \cos 2\left(\alpha + 150^\circ\right)\right] \end{cases}$$

$$\begin{cases} m_0 = w^2 \left[ \lambda_{m\sigma} + \frac{1}{4}\left(\lambda_{ad} + \lambda_{aq}\right)\right] \\ m_2 = \frac{1}{2} w^2 \left(\lambda_{ad} - \lambda_{aq}\right), \quad w_a = w_b = w \end{cases}$$

**隐极同步电机：$m_2$=0，$L_{ab}$= $L_{bc}$= $L_{ca}$=const**

# (3)转子各绕组的自感系数和互感系数

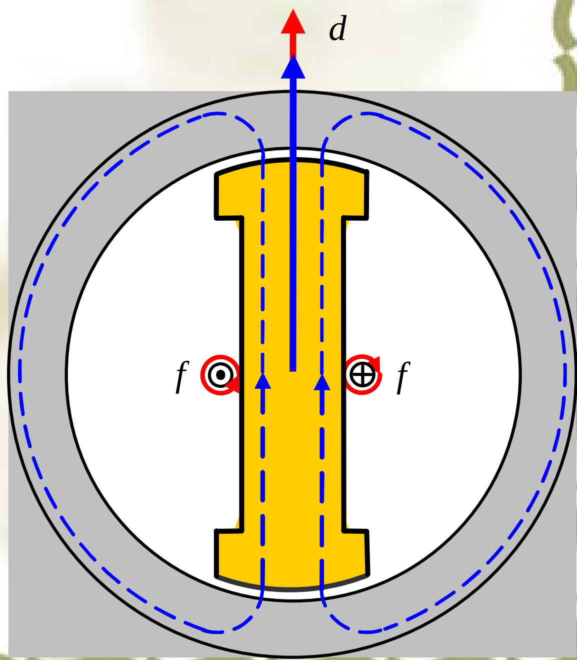



绕组轴线正向示意图

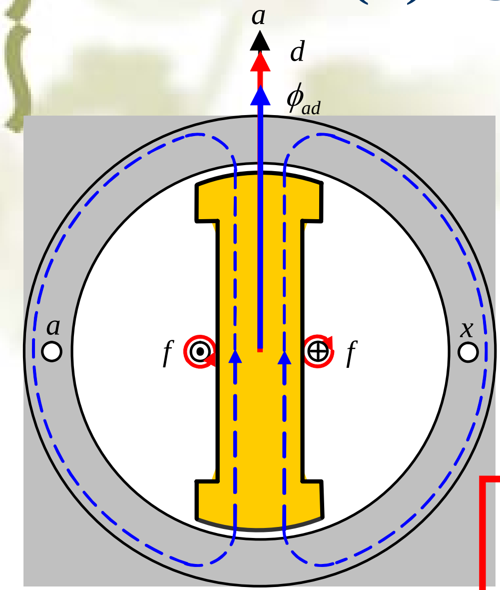
$$
\begin{cases}
L_{ff} = L_f = w_f^2\left(\lambda_{\sigma f} + \lambda_{ad}\right) \\
L_{DD} = L_D = w_D^2\left(\lambda_{\sigma D} + \lambda_{ad}\right) \\
L_{QQ} = L_Q = w_Q^2\left(\lambda_{\sigma Q} + \lambda_{aq}\right)
\end{cases}
$$

$$
\begin{cases}
L_{fD} = L_{Df} = const \\
L_{fQ} = L_{Qf} = 0 \\
L_{DQ} = L_{QD} = 0
\end{cases}
$$

$$
\begin{cases}
\psi_{ff} = w_f^2 i_f\left(\lambda_{\sigma f} + \lambda_{ad}\right) \\
L_f = w_f^2\left(\lambda_{\sigma f} + \lambda_{ad}\right)
\end{cases}
$$

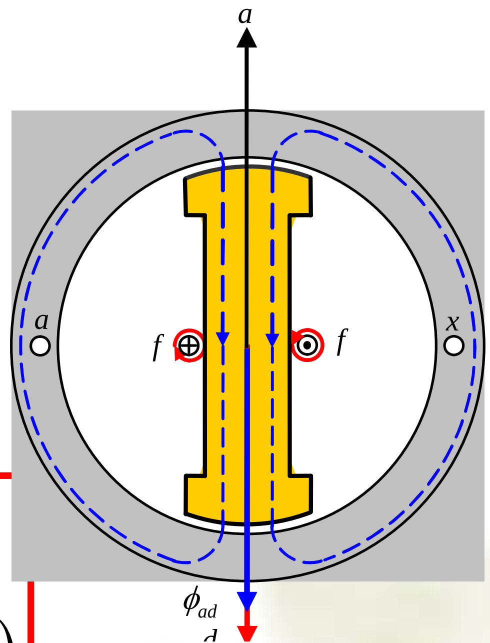


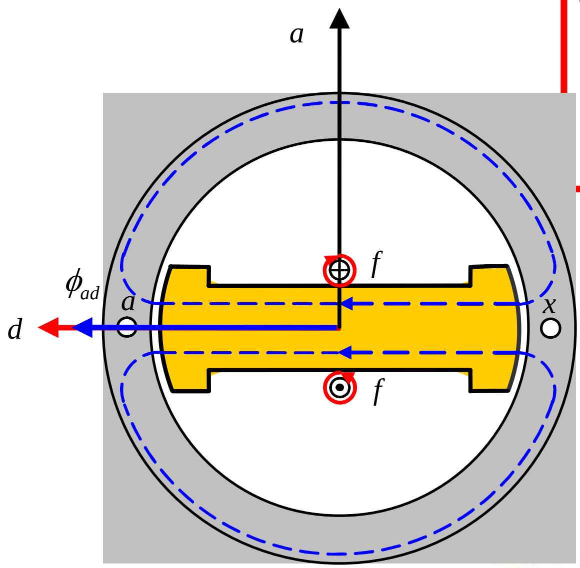
# (4)定子绕组和转子各绕组间的互感系数—abc--*f*

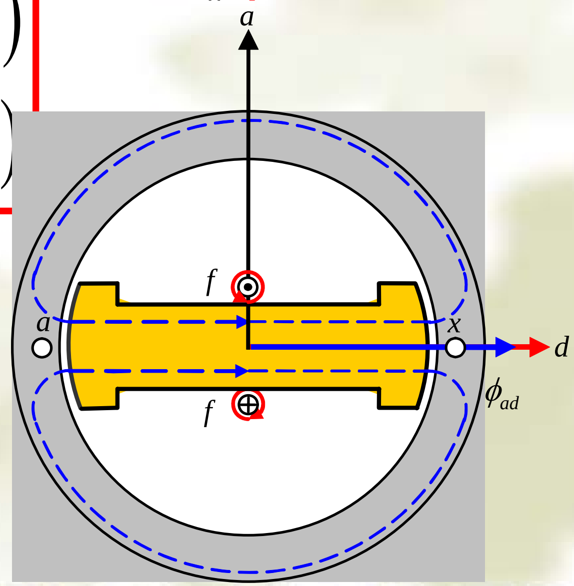



$$\psi_{af} = ww_f i_f \lambda_{ad} \cos\alpha$$

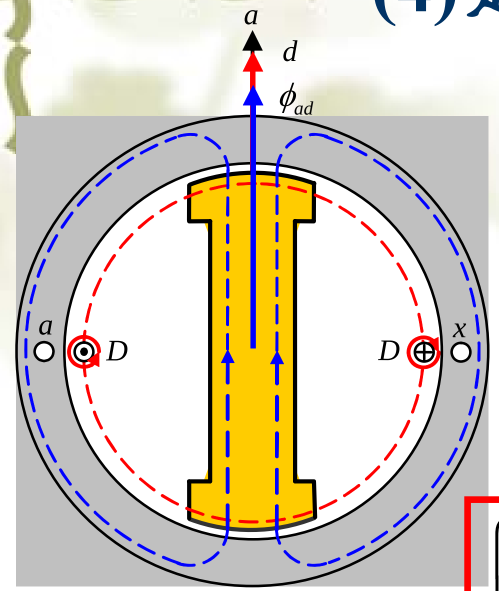
$$\begin{cases} L_{fa} = L_{af} = m_{af} \cos\alpha \\ L_{fb} = L_{bf} = m_{af} \cos\left(\alpha - 120^\circ\right) \\ L_{fc} = L_{cf} = m_{af} \cos\left(\alpha + 120^\circ\right) \end{cases}$$



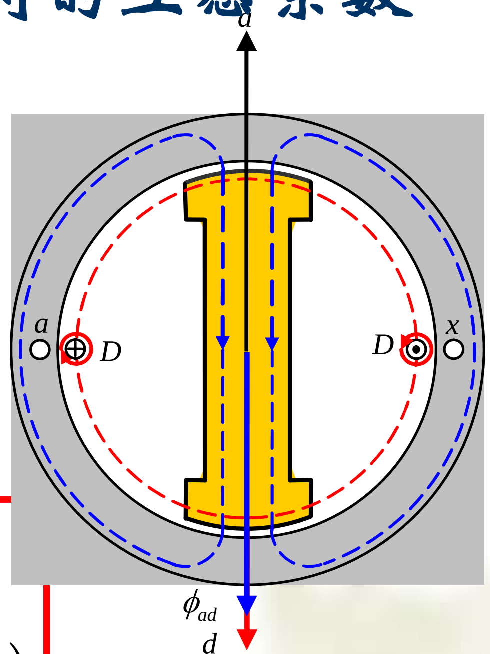



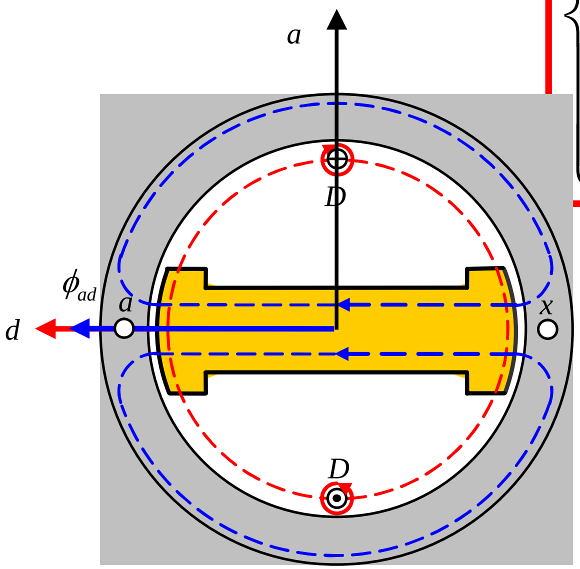
## (4)定子绕组和转子各绕组间的互感系数—abc--*D*

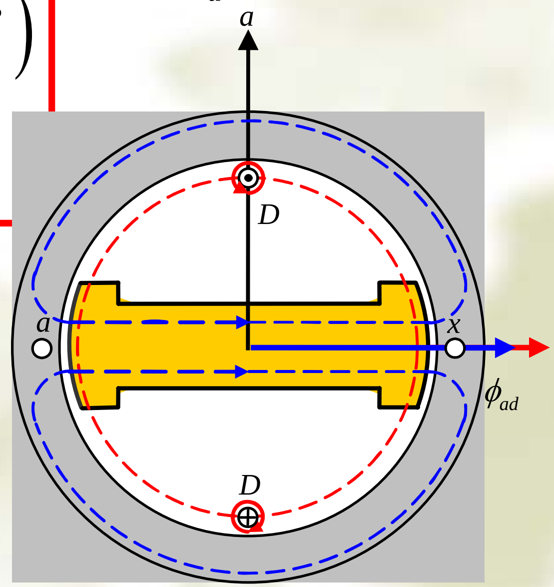



$$\psi_{aD} = w w_D i_D \lambda_{ad} \cos\alpha$$

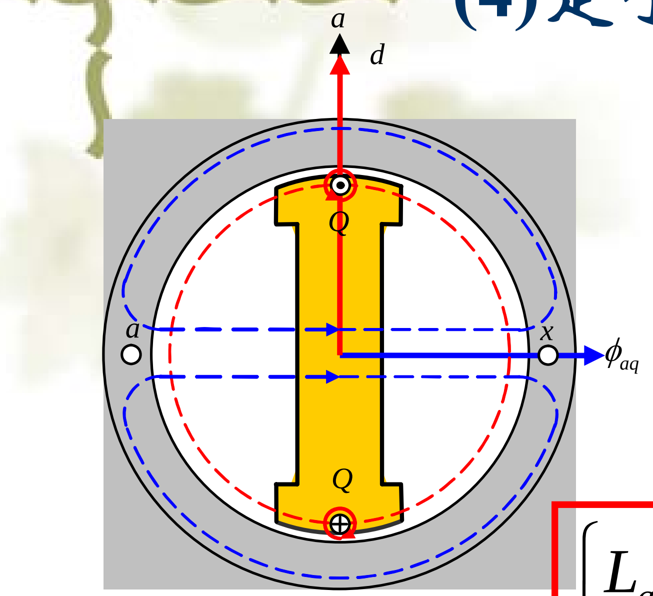



$$\begin{cases} L_{aD} = L_{Da} = m_{aD} \cos\alpha \\ L_{bD} = L_{Db} = m_{aD} \cos\left(\alpha - 120^\circ\right) \\ L_{cD} = L_{Dc} = m_{aD} \cos\left(\alpha + 120^\circ\right) \end{cases}$$

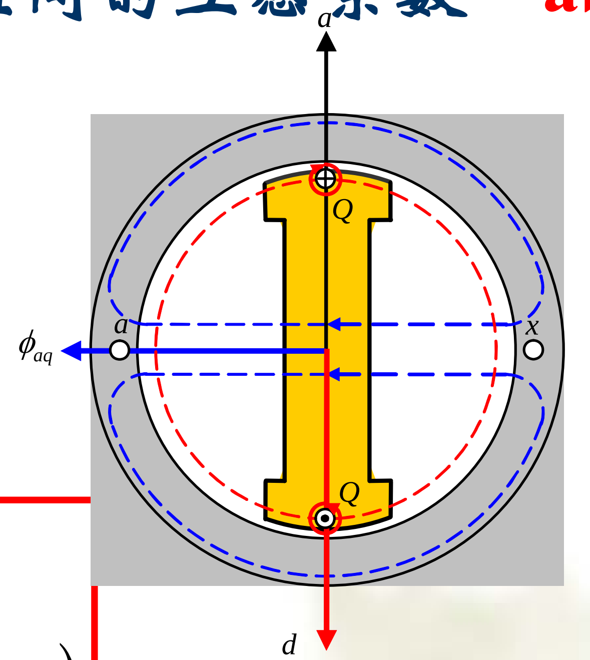


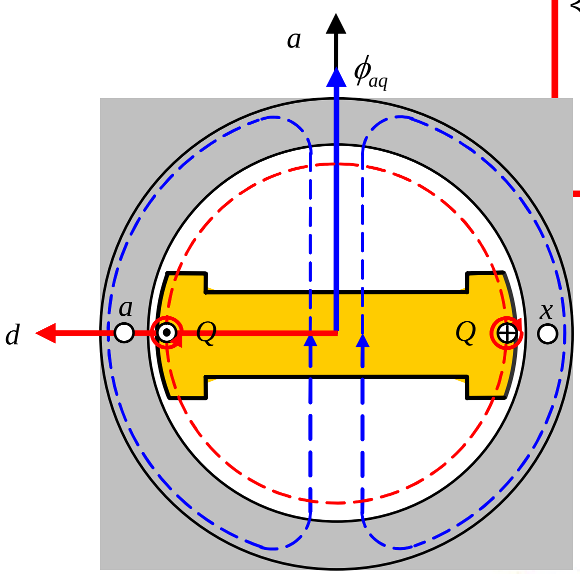
## (4)定子绕组和转子各绕组间的互感系数—abc--*Q*

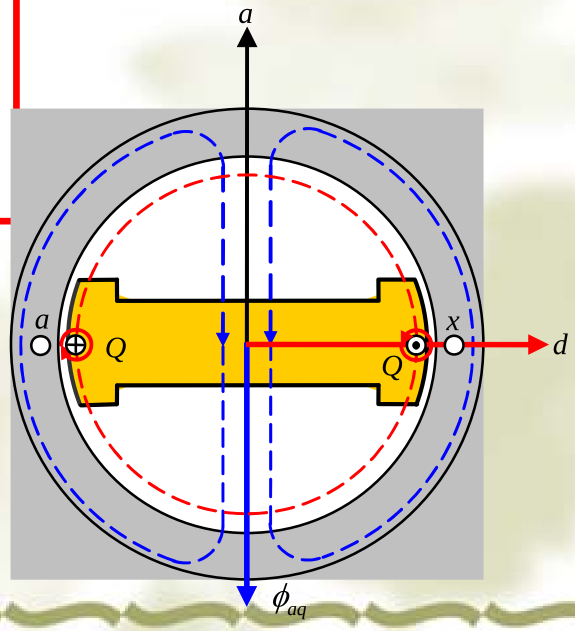



$$\psi_{aQ} = w w_Q i_Q \lambda_{aq} \sin\alpha$$




$$\begin{cases} L_{aQ} = L_{Qa} = m_{aQ} \sin\alpha \\ L_{bQ} = L_{Qb} = m_{aQ} \sin\left(\alpha - 120^\circ\right) \\ L_{cQ} = L_{Qc} = m_{aQ} \sin\left(\alpha + 120^\circ\right) \end{cases}$$



# 本节主要结论

磁链方程式中，同步电机许多电感系数随转子位置角发生周期性变化，是时变系数

将磁链方程代入同步电机电势方程，将得到一组时变系数微分方程，不便于求解；

| | 凸极机 | 隐极机 | 影响因素 |
|---|---|---|---|
| 定子绕组自感系数 | 时变 | 常数 | 转子dq轴向对称性 |
| 定子绕组间互感系数 | 时变 | 常数 | 转子dq轴向对称性 |
| 转子绕组自感系数 | 常数 | 常数 | 磁路恒定 |
| 转子绕组间互感系数 | 常数 | 常数 | 磁路恒定 |
| 定转子绕组间互感系数 | 时变 | 时变 | 定转子相对运动 |

## 磁链方程式出现变系数的原因：

（1）转子的旋转使定转子绕组间产生相对运动，致使定转子绕组间的互感系数发生相应的周期性变化；

（2）转子在磁路上只是分别对d轴和q轴对称，而不是随意对称，由此导致定子各绕组的自感和互感发生周期性变化；

# 3-3 dq0坐标系的同步电机方程

1. 坐标变换和dq0坐标系

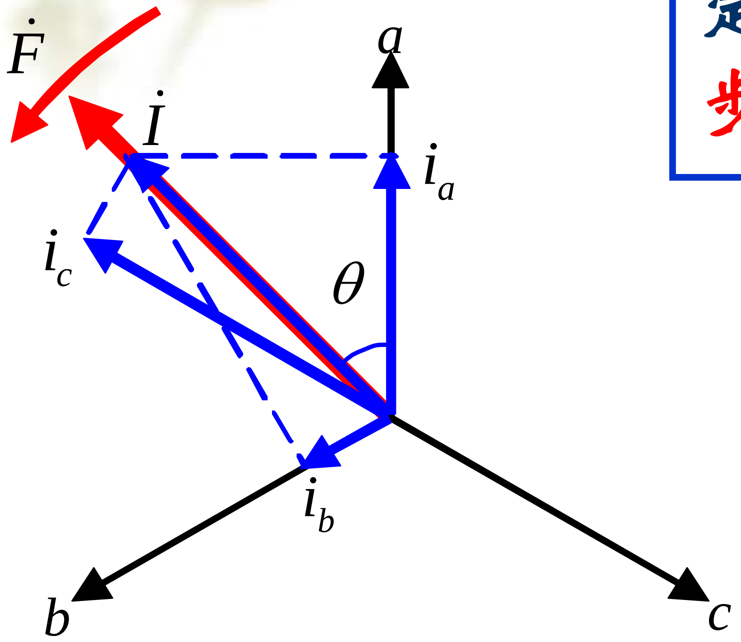
2. dq0坐标系下的电势方程

3. dq0坐标系下的磁链方程和电感系数

4. 同步电机标幺值基本方程

## 1. 坐标变换与dq0坐标系

### (1) 采用通用相量表示定子三相电流



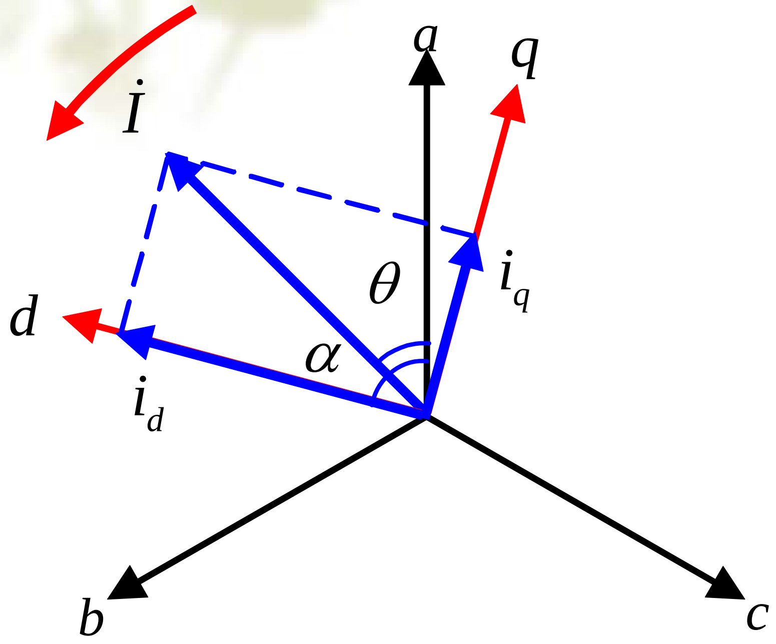

定子三相对称电流可以用以同步转速旋转的通用相量$I$表示；

$$\begin{cases} i_a = I\cos\theta \\ i_b = I\cos(\theta - 120^\circ) \\ i_c = I\cos(\theta + 120^\circ) \end{cases}$$

注：定子三相对称电流产生空间同步旋转的电枢磁势$F$，可以视为由通用相量$I$产生，两者同相位，数值成比例；

## 1. 坐标变换与dq0坐标系

### （2）通用相量的dq轴分量



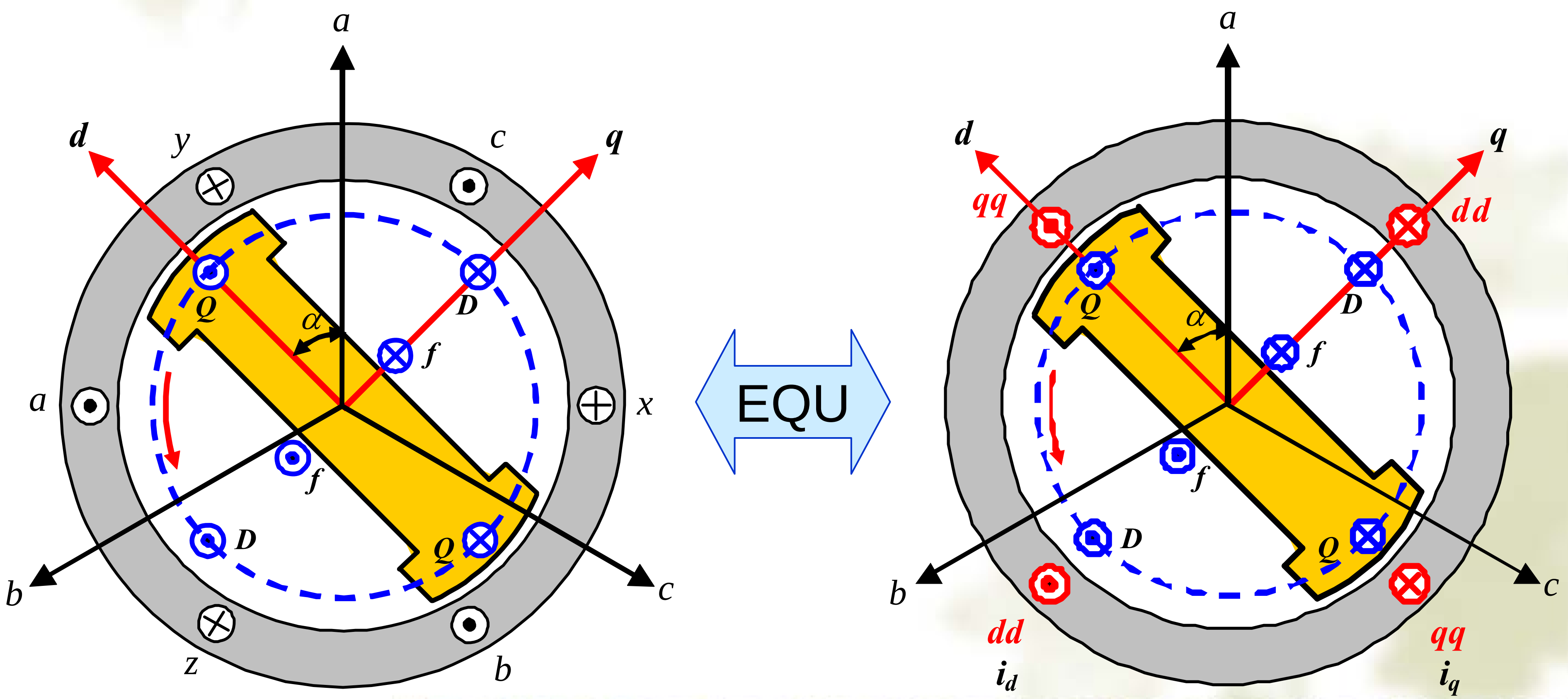

以同步转速旋转的**通用相量*I***可以分解为两个同步旋转的分量$i_d$和$i_q$；

$$\begin{cases} i_d = I\cos(\alpha-\theta) \\ i_q = I\sin(\alpha-\theta) \end{cases}$$

注：定子三相对称电流产生空间同步旋转的**电枢磁势*F***，亦可以视为由**通用相量*I***的分量$i_d$和$i_q$产生；

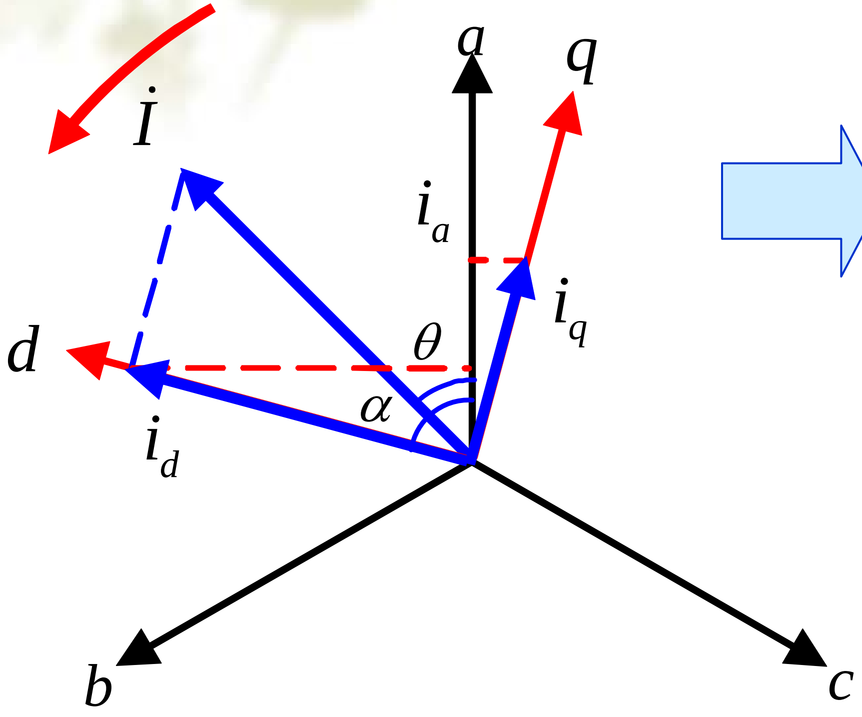
# 1. 坐标变换与dq0坐标系—Park变换

设想：将静止的abc三相定子绕组等效为随转子旋转的dd和qq绕组。等效绕组中的电流$i_d$和$i_q$产生的磁势对转子相对静止，磁通磁路磁阻不变，因此电感系数为常数。



## 1. 坐标变换与dq0坐标系

### (3) 用dq轴分量表示$i_{abc}$




$$\begin{cases} i'_a = i_d \cos\alpha + i_q \sin\alpha \\ i'_b = i_d \cos(\alpha - 120^\circ) + i_q \sin(\alpha - 120^\circ) \\ i'_c = i_d \cos(\alpha + 120^\circ) + i_q \sin(\alpha + 120^\circ) \end{cases}$$

计及三相不平衡，增加零轴分量

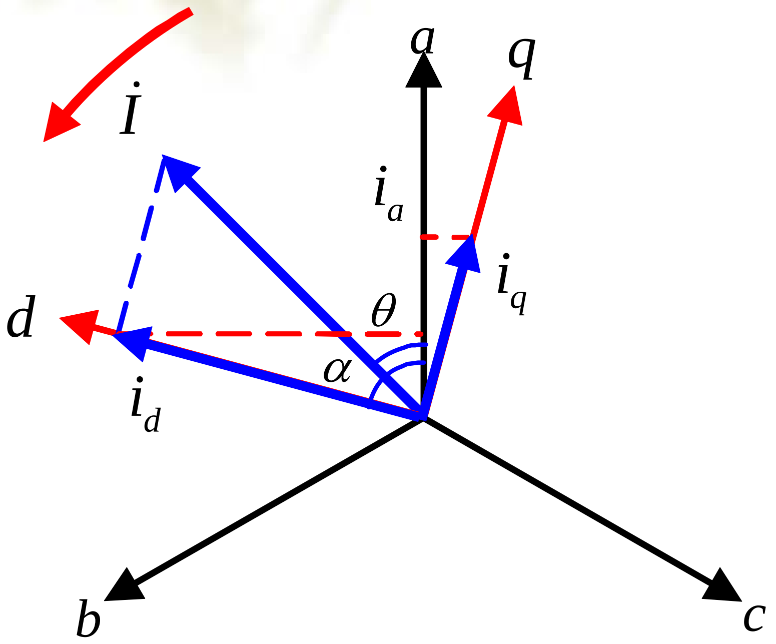
$$i_0 = \frac{1}{3}(i_a + i_b + i_c)$$

$$\begin{cases} i_a = i'_a + i_0 \\ i_b = i'_b + i_0 \\ i_c = i'_c + i_0 \end{cases}$$

## 1. 坐标变换与dq0坐标系

### (3) Park变换—$\mathbf{i}_{dq0}$---$\mathbf{i}_{abc}$

计及三相不平衡，增加零轴分量




$$i_0 = \frac{1}{3}(i_a + i_b + i_c)$$

$$\begin{cases} i_a = i_d \cos\alpha + i_q \sin\alpha + i_0 \\ i_b = i_d \cos(\alpha - 120^\circ) + i_q \sin(\alpha - 120^\circ) + i_0 \\ i_c = i_d \cos(\alpha + 120^\circ) + i_q \sin(\alpha + 120^\circ) + i_0 \end{cases}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos(\alpha - 120^\circ) & \sin(\alpha - 120^\circ) & 1 \\ \cos(\alpha + 120^\circ) & \sin(\alpha + 120^\circ) & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}$$

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\mathbf{i}_{dq0}$$

## 1. 坐标变换与dq0坐标系

### (3) Park变换— $\mathbf{i}_{abc}$-----$\mathbf{i}_{dq0}$

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\mathbf{i}_{dq0}$$

$$\mathbf{i}_{dq0} = \mathbf{P}\mathbf{i}_{abc}$$

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ) \\ \sin\alpha & \sin(\alpha-120^\circ) & \sin(\alpha+120^\circ) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

注：从旋转的dq0坐标系观察到的定子电流为：$\mathbf{i}_{dq0}$

## 1. 坐标变换与dq0坐标系

### Park变换的另一种推导方法

$$\begin{cases} i_a = I\cos\theta \\ i_b = I\cos(\theta - 120^\circ) \\ i_c = I\cos(\theta + 120^\circ) \end{cases}$$

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ) \\ \sin\alpha & \sin(\alpha-120^\circ) & \sin(\alpha+120^\circ) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\begin{cases} i_d = I\cos(\alpha - \theta) \\ i_q = I\sin(\alpha - \theta) \end{cases}$$

$$i_0 = \frac{1}{3}(i_a + i_b + i_c)$$

$$\begin{cases} \cos(\alpha-\theta) = \frac{2}{3}\left[\cos\alpha\cos\theta + \cos(\alpha-120^\circ)\cos(\theta-120^\circ) + \cos(\alpha+120^\circ)\cos(\theta+120^\circ)\right] \\ \sin(\alpha-\theta) = \frac{2}{3}\left[\sin\alpha\cos\theta + \sin(\alpha-120^\circ)\cos(\theta-120^\circ) + \sin(\alpha+120^\circ)\cos(\theta+120^\circ)\right] \end{cases}$$

# 1. 坐标变换与dq0坐标系

同理可对定子电压和磁链作同样的变换。

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\mathbf{i}_{dq0}$$

$$\mathbf{u}_{abc} = \mathbf{P}^{-1}\mathbf{u}_{dq0}$$

$$\boldsymbol{\psi}_{abc} = \mathbf{P}^{-1}\boldsymbol{\psi}_{dq0}$$

$$\mathbf{P}^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos(\alpha-120^\circ) & \sin(\alpha-120^\circ) & 1 \\ \cos(\alpha+120^\circ) & \sin(\alpha+120^\circ) & 1 \end{bmatrix}$$

$$\mathbf{i}_{dq0} = \mathbf{P}\mathbf{i}_{abc}$$

$$\mathbf{u}_{dq0} = \mathbf{P}\mathbf{u}_{abc}$$

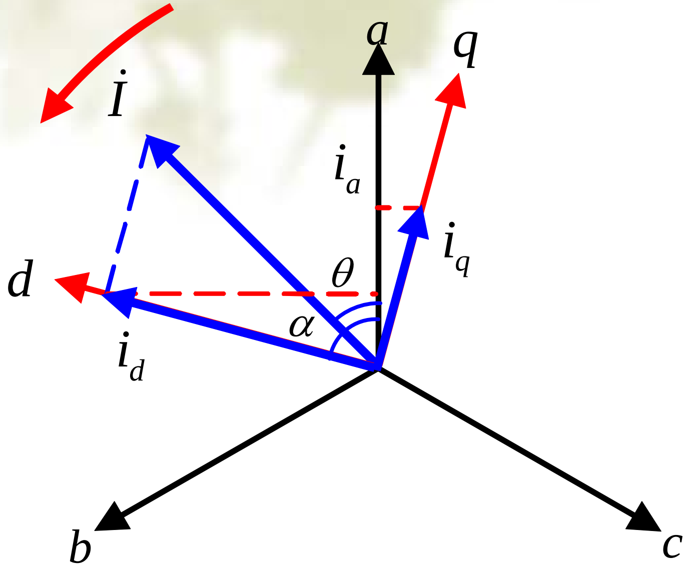
$$\boldsymbol{\psi}_{dq0} = \mathbf{P}\boldsymbol{\psi}_{abc}$$

$$\mathbf{P} = \frac{2}{3}\begin{bmatrix} \cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ) \\ \sin\alpha & \sin(\alpha-120^\circ) & \sin(\alpha+120^\circ) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

需要注意的是：Park变换本质上是数学变换，因此其矩阵形式不是唯一的。

## 1. 坐标变换与dq0坐标系

### （4）不同频率abc三相对称电流的dq0分量




| abc对称分量 | | dq0分量 | |
|---|---|---|---|
| 直流电流 | $\mathbf{i}_{abc}(\omega'=0)$ | 基频电流 | $\mathbf{i}_{dq0}(\omega)$ |
| 基频交流 | $\mathbf{i}_{abc}(\omega'=\omega)$ | 直流电流 | $\mathbf{i}_{dq0}(0)$ |
| 倍频交流 | $\mathbf{i}_{abc}(\omega'=2\omega)$ | 基频交流 | $\mathbf{i}_{dq0}(\omega)$ |

P

$$\begin{cases} i_a = I\cos\theta \\ i_b = I\cos(\theta-120^\circ) \\ i_c = I\cos(\theta+120^\circ) \end{cases}$$

$$\begin{cases} i_d = I\cos(\alpha-\theta) = I\cos\left[(\alpha_0-\theta_0)+(\omega'-\omega)t\right] \\ i_q = I\sin(\alpha-\theta) = I\sin\left[(\alpha_0-\theta_0)+(\omega'-\omega)t\right] \end{cases}$$

$$\theta = \omega' t+\theta_0, \quad \alpha = \omega t+\alpha_0$$

## 2. dq0坐标系下的发电机电势方程

Park变换仅对定子绕组电势方程和磁链方程

$\mathbf{u}_{dq0} = \mathbf{P}\mathbf{u}_{abc} \quad \boldsymbol{\psi}_{dq0} = \mathbf{P}\boldsymbol{\psi}_{abc} \quad \mathbf{i}_{dq0} = \mathbf{P}\mathbf{i}_{abc}$

$\mathbf{u}_{abc} = -\dot{\boldsymbol{\psi}}_{abc} - \mathbf{r}_S\mathbf{i}_{abc}$ $\xrightarrow{\mathbf{P}\times}$ $\mathbf{P}\mathbf{u}_{abc} = -\mathbf{P}\dot{\boldsymbol{\psi}}_{abc} - \mathbf{r}_S\mathbf{P}\mathbf{i}_{abc}$

$\dot{\boldsymbol{\psi}}_{dq0} = \frac{d}{dt}\left[\mathbf{P}\boldsymbol{\psi}_{abc}\right] = \mathbf{P}\dot{\boldsymbol{\psi}}_{abc} + \dot{\mathbf{P}}\boldsymbol{\psi}_{abc}$ $\Rightarrow$ $\mathbf{P}\dot{\boldsymbol{\psi}}_{abc} = \dot{\boldsymbol{\psi}}_{dq0} - \dot{\mathbf{P}}\mathbf{P}^{-1}\boldsymbol{\psi}_{dq0}$

$\mathbf{u}_{dq0} = -\left(\dot{\boldsymbol{\psi}}_{dq0} + \mathbf{S}\right) - \mathbf{r}_S\mathbf{i}_{dq0}$ $\Rightarrow$

$$\begin{cases} u_d = -\dot{\psi}_d - \omega\psi_q - r\,i_d \\ u_q = -\dot{\psi}_q + \omega\psi_d - r\,i_q \\ u_0 = -\dot{\psi}_0 - r\,i_0 \end{cases}$$

$\mathbf{S} = -\dot{\mathbf{P}}\mathbf{P}^{-1}\boldsymbol{\psi}_{dq0}$

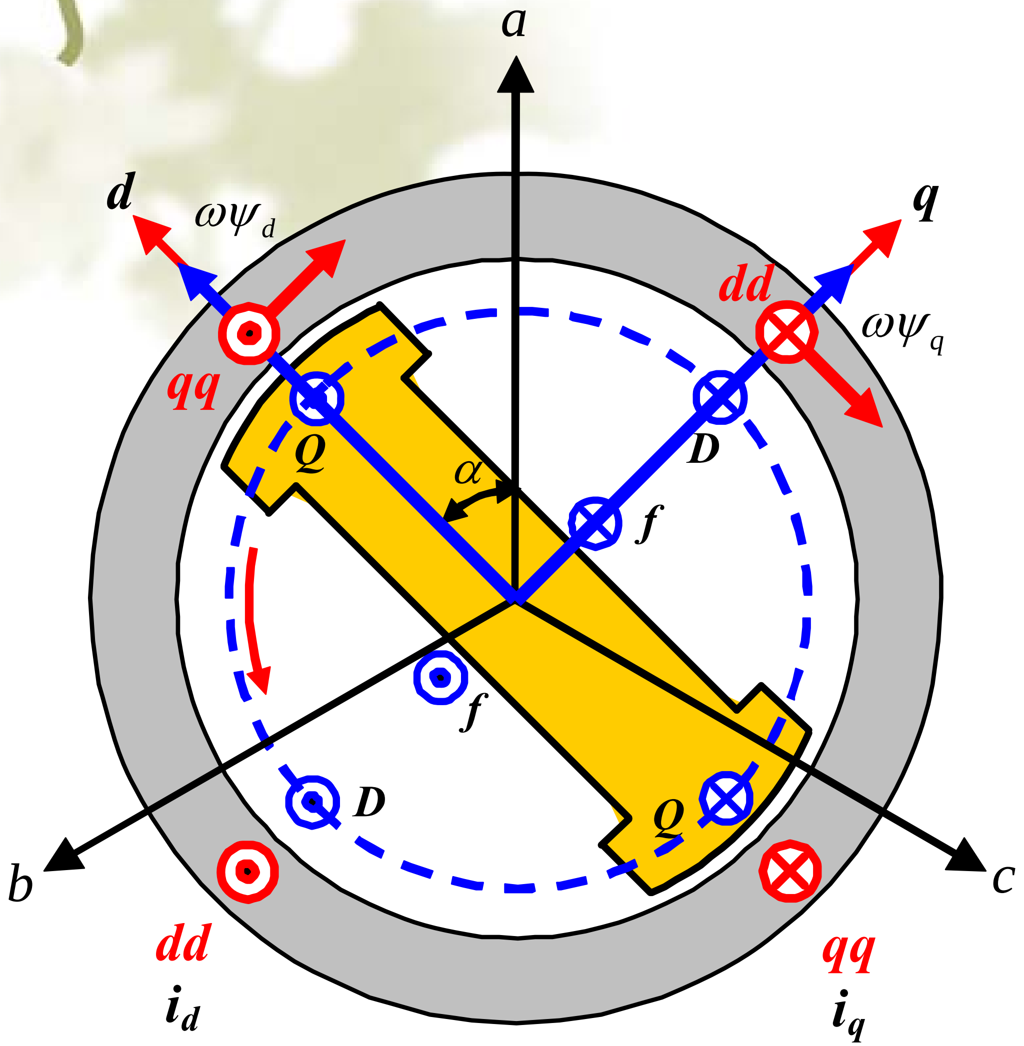
## 2. dq0坐标系下的电势方程

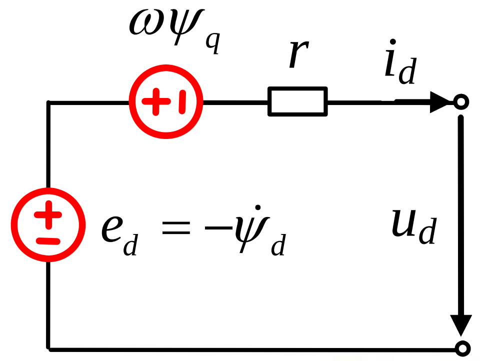
$$\dot{\mathbf{P}}=\frac{2}{3}\frac{d}{dt}\begin{bmatrix}\cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ)\\ \sin\alpha & \sin(\alpha-120^\circ) & \sin(\alpha+120^\circ)\\ 1/2 & 1/2 & 1/2\end{bmatrix}=\frac{2}{3}\frac{d\alpha}{dt}\begin{bmatrix}-\sin\alpha & -\sin(\alpha-120^\circ) & -\sin(\alpha+120^\circ)\\ \cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ)\\ 0 & 0 & 0\end{bmatrix}$$

$$\dot{\mathbf{P}}\mathbf{P}^{-1}=\frac{2}{3}\frac{d\alpha}{dt}\begin{bmatrix}-\sin\alpha & -\sin(\alpha-120^\circ) & -\sin(\alpha+120^\circ)\\ \cos\alpha & \cos(\alpha-120^\circ) & \cos(\alpha+120^\circ)\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 1\\ \cos(\alpha-120^\circ) & \sin(\alpha-120^\circ) & 1\\ \cos(\alpha+120^\circ) & \sin(\alpha+120^\circ) & 1\end{bmatrix}$$

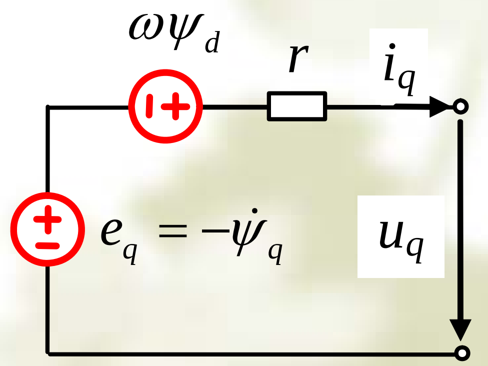
$$\mathbf{S}=-\dot{\mathbf{P}}\mathbf{P}^{-1}\boldsymbol{\psi}_{dq0}=-\frac{2}{3}\begin{bmatrix}0 & -\frac{3}{2}\frac{d\alpha}{dt} & 0\\ \frac{3}{2}\frac{d\alpha}{dt} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_0\end{bmatrix}=\begin{bmatrix}\omega\psi_q\\ -\omega\psi_d\\ 0\end{bmatrix}$$

# 2. dq0坐标系的电势方程——“伪静止”等效绕组




$$u_d = -\dot{\psi}_d - \omega\psi_q - ri_d$$

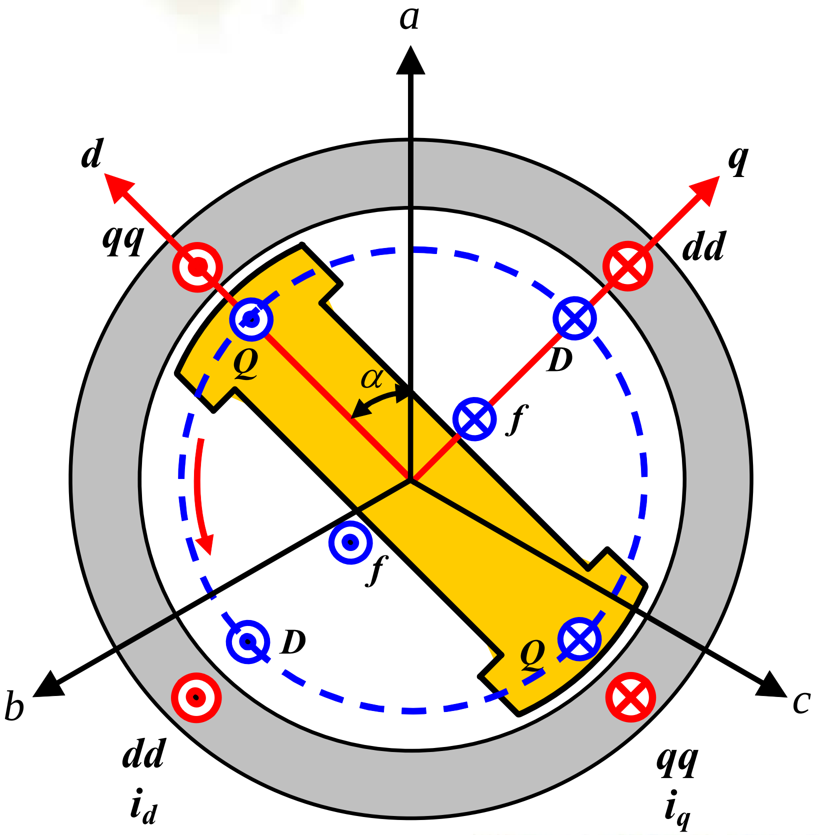



$$u_q = -\dot{\psi}_q + \omega\psi_d - ri_q$$

# 3. dq0坐标系下的磁链方程及等值电感系数

$$\boldsymbol{\psi}_{abc} = \mathbf{L}_{SS}\mathbf{i}_{abc} + \mathbf{L}_{SR}\mathbf{i}_{fDQ}$$
$$\boldsymbol{\psi}_{fDQ} = \mathbf{L}_{RS}\mathbf{i}_{abc} + \mathbf{L}_{RR}\mathbf{i}_{fDQ}$$

$$\mathbf{P}\boldsymbol{\psi}_{abc} = \mathbf{P}\mathbf{L}_{SS}\mathbf{P}^{-1}\mathbf{P}\mathbf{i}_{abc} + \mathbf{P}\mathbf{L}_{SR}\mathbf{i}_{fDQ}$$
$$\boldsymbol{\psi}_{fDQ} = \mathbf{L}_{RS}\mathbf{P}^{-1}\mathbf{P}\mathbf{i}_{abc} + \mathbf{L}_{RR}\mathbf{i}_{fDQ}$$




$$\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_0\\ \psi_f\\ \psi_D\\ \psi_Q\end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & m_{af} & m_{aD} & 0\\ 0 & L_q & 0 & 0 & 0 & m_{aQ}\\ 0 & 0 & L_0 & 0 & 0 & 0\\ \frac{3}{2}m_{fa} & 0 & 0 & L_f & L_{fD} & 0\\ \frac{3}{2}m_{Da} & 0 & 0 & L_{Df} & L_D & 0\\ 0 & \frac{3}{2}m_{Qa} & 0 & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_d\\ i_q\\ i_0\\ i_f\\ i_D\\ i_Q\end{bmatrix}$$

# 3-4 同步电机常用标幺制-同步电机标幺值方程

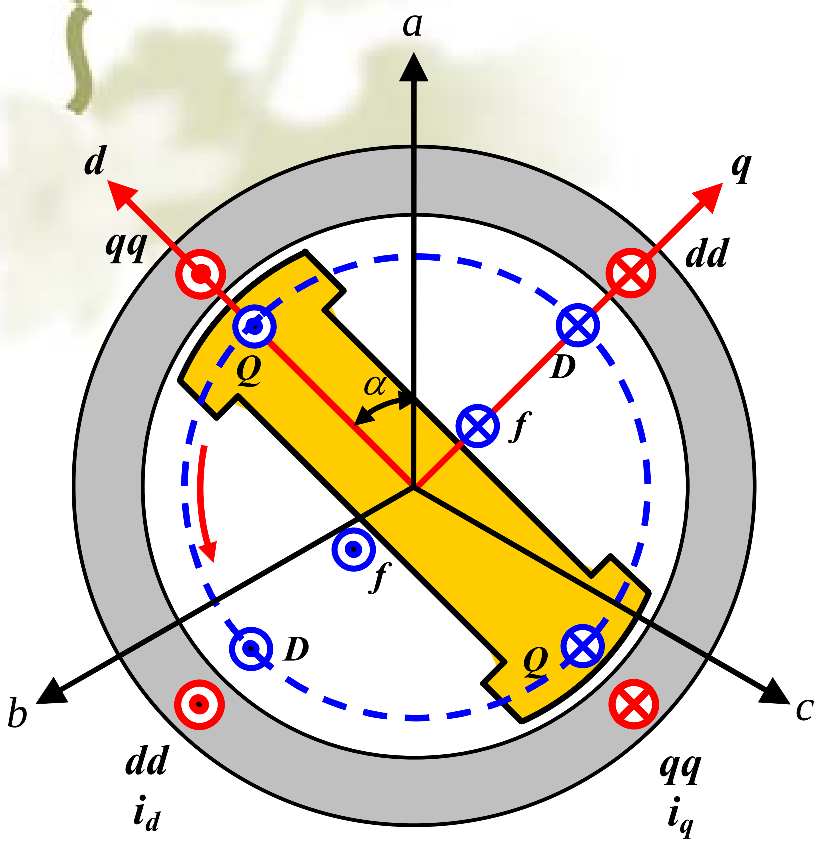
$$\begin{cases} u_d = -\dot{\psi}_d - \omega\psi_q - r\,i_d \\ u_q = -\dot{\psi}_q + \omega\psi_d - r\,i_q \\ u_0 = -\dot{\psi}_0 - r\,i_0 \end{cases}$$

$$\begin{cases} -u_f = -\dot{\psi}_f - r_f i_f \\ 0 \quad = -\dot{\psi}_D - r_D i_D \\ 0 \quad = -\dot{\psi}_Q - r_Q i_Q \end{cases}$$

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & m_{af} & m_{aD} & 0 \\ 0 & L_q & 0 & 0 & 0 & m_{aQ} \\ 0 & 0 & L_0 & 0 & 0 & 0 \\ m_{fa} & 0 & 0 & L_f & L_{fD} & 0 \\ m_{Da} & 0 & 0 & L_{Df} & L_D & 0 \\ 0 & m_{Qa} & 0 & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

$$\begin{aligned} L_d &= x_d \\ L_q &= x_q \\ L_0 &= x_0 \\ L_f &= x_f \\ L_D &= x_D \\ L_Q &= x_Q \\ m_{af} &= x_{af} \\ m_{aD} &= x_{aD} \\ L_{fD} &= x_{fD} \\ m_{aQ} &= x_{aQ} \end{aligned}$$

# 3-5 基本方程的拉氏运算式-同步电机的电抗



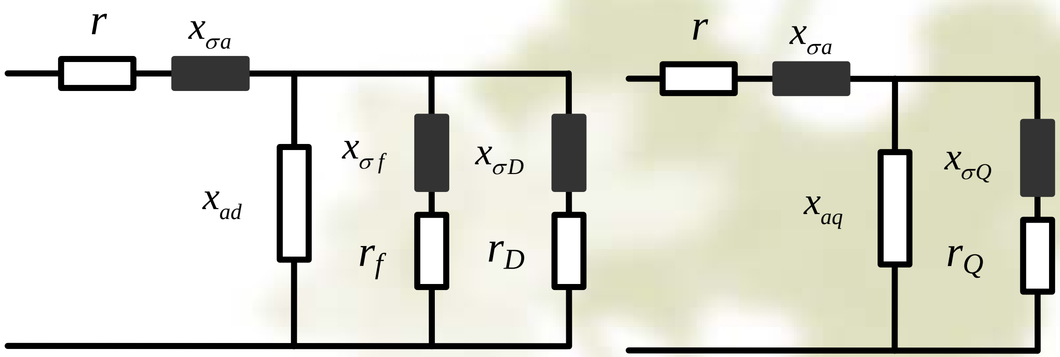

基于公共耦合磁通的假设

$$x_{af} = x_{aD} = x_{fD} = x_{ad}, \quad m_{aQ} = x_{aQ} = x_{aq}$$

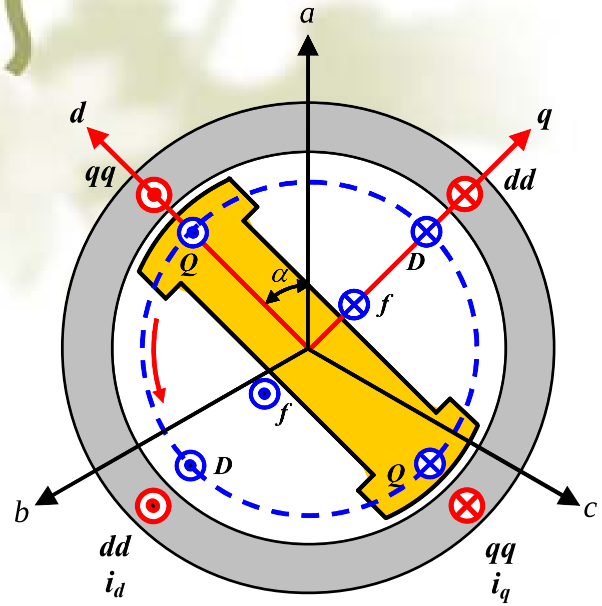
$$\begin{cases} x_d = x_{\sigma a} + x_{ad} \\ x_f = x_{\sigma f} + x_{ad} \\ x_D = x_{\sigma D} + x_{ad} \end{cases}$$

$$\begin{cases} x_q = x_{\sigma a} + x_{aq} \\ x_Q = x_{\sigma Q} + x_{aq} \end{cases}$$

与变压器等值电路类似



# 3-5 用电抗表示的同步电机标幺值方程




$$\begin{cases} u_d = -\dot{\psi}_d - \omega\psi_q - r\,i_d \\ u_q = -\dot{\psi}_q + \omega\psi_d - r\,i_q \\ u_0 = -\dot{\psi}_0 - r\,i_0 \end{cases}$$

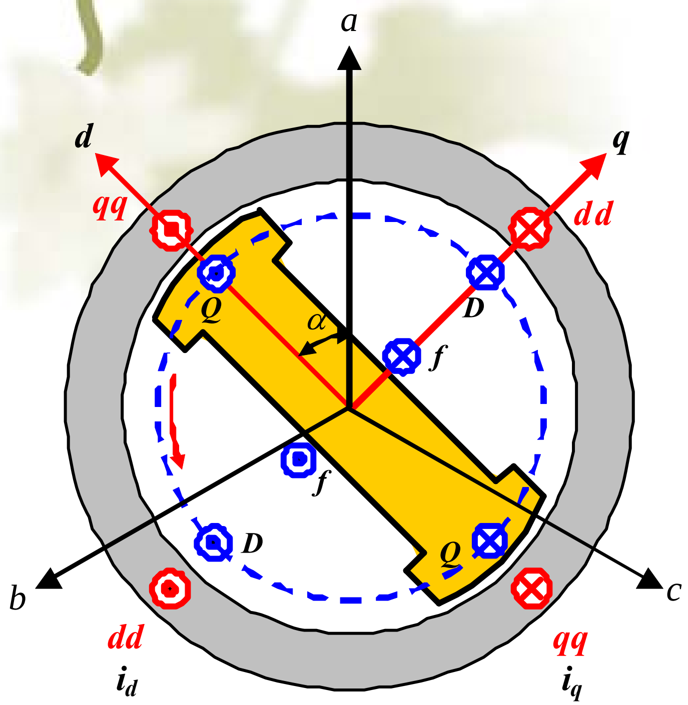
$$\begin{cases} -u_f = -\dot{\psi}_f - r_f i_f \\ 0 = -\dot{\psi}_D - r_D i_D \\ 0 = -\dot{\psi}_Q - r_Q i_Q \end{cases}$$

$$\begin{cases} x_d = x_{\sigma a} + x_{ad} \\ x_f = x_{\sigma f} + x_{ad} \\ x_D = x_{\sigma D} + x_{ad} \end{cases}$$

$$\begin{cases} x_q = x_{\sigma a} + x_{aq} \\ x_Q = x_{\sigma Q} + x_{aq} \end{cases}$$

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} x_d & 0 & 0 & x_{ad} & x_{ad} & 0 \\ 0 & x_q & 0 & 0 & 0 & x_{aq} \\ 0 & 0 & x_0 & 0 & 0 & 0 \\ x_{ad} & 0 & 0 & x_f & x_{ad} & 0 \\ x_{ad} & 0 & 0 & x_{ad} & x_D & 0 \\ 0 & x_{aq} & 0 & 0 & 0 & x_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

## 3-5 同步电机标幺值方程—矩阵形式




$$\begin{bmatrix} u_d \\ -u_f \\ 0 \end{bmatrix} = -\begin{bmatrix} \dot{\psi}_d \\ \dot{\psi}_f \\ \dot{\psi}_D \end{bmatrix} - \begin{bmatrix} \omega\psi_q \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} r & 0 & 0 \\ 0 & r_f & 0 \\ 0 & 0 & r_D \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix}$$

$$\begin{bmatrix} u_q \\ 0 \end{bmatrix} = -\begin{bmatrix} \dot{\psi}_q \\ \dot{\psi}_Q \end{bmatrix} + \begin{bmatrix} \omega\psi_d \\ 0 \end{bmatrix} - \begin{bmatrix} r & 0 \\ 0 & r_Q \end{bmatrix} \begin{bmatrix} i_q \\ i_Q \end{bmatrix}$$

$$u_0 = -\dot{\psi}_0 - r i_0$$

$$\psi_0 = x_0 i_0$$

$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_D \end{bmatrix} = \begin{bmatrix} x_d & x_{ad} & x_{ad} \\ x_{ad} & x_f & x_{ad} \\ x_{ad} & x_{ad} & x_D \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix}$$

$$\begin{bmatrix} \psi_q \\ \psi_Q \end{bmatrix} = \begin{bmatrix} x_q & x_{aq} \\ x_{aq} & x_Q \end{bmatrix} \begin{bmatrix} i_q \\ i_Q \end{bmatrix}$$

# 同步电机电磁功率

## 电磁功率有名值计算式

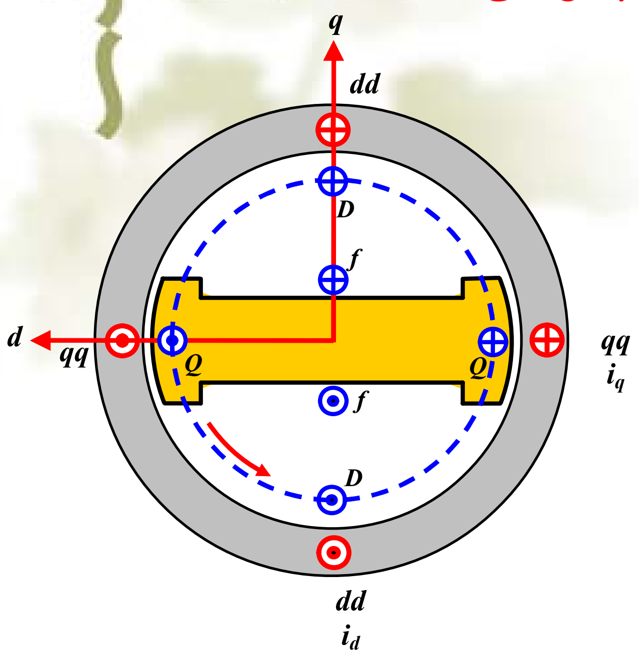
$$P = \mathbf{u}_{abc}^{\mathrm{T}} \mathbf{i}_{abc} = \left[\mathbf{P}^{-1}\mathbf{u}_{dq0}\right]^{\mathrm{T}} \mathbf{P}^{-1}\mathbf{i}_{dq0}$$

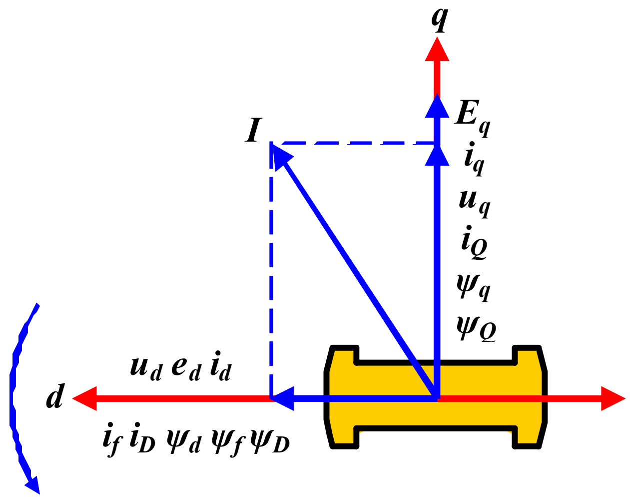
$$= \mathbf{u}_{dq0}^{\mathrm{T}} \left[\mathbf{P}^{-1}\right]^{\mathrm{T}} \mathbf{P}^{-1}\mathbf{i}_{dq0} = 3u_0 i_0 + \frac{3}{2}\left(u_d i_d + u_q i_q\right)$$

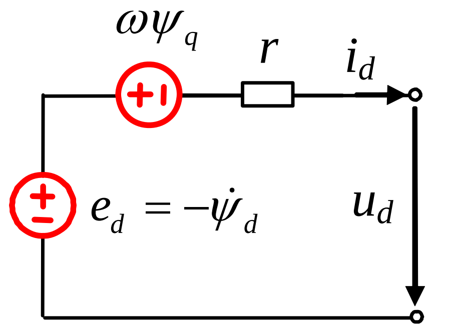
## 电磁功率标幺值计算式

$$P = 2u_0 i_0 + u_d i_d + u_q i_q$$

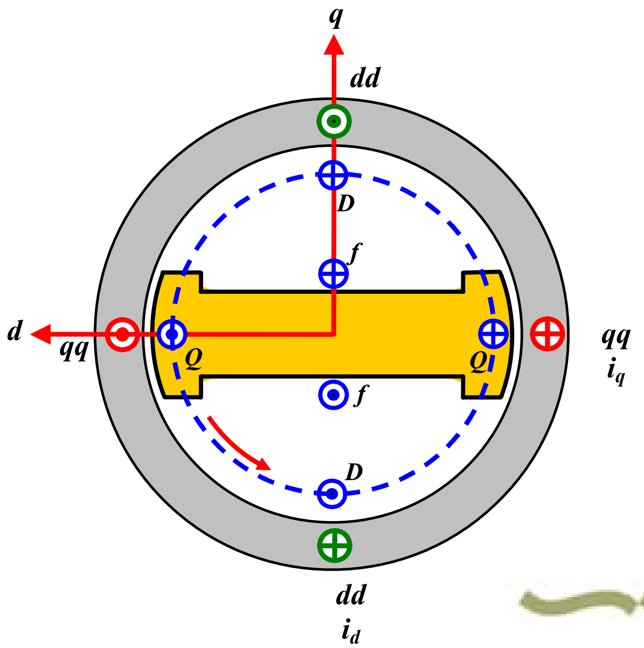
# 3-6 同步电机的对称稳态运行-实用正方向

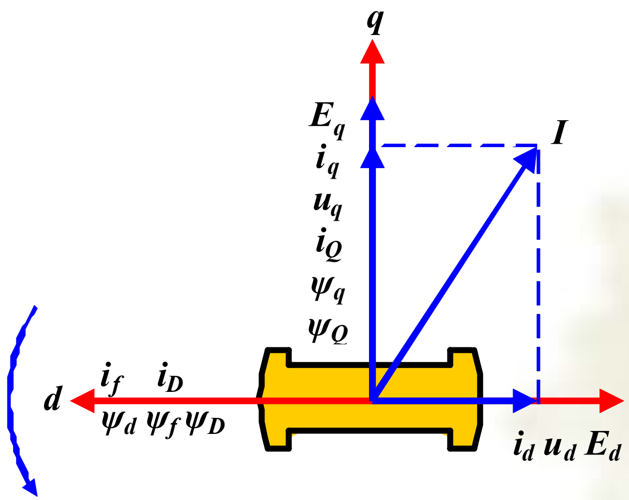


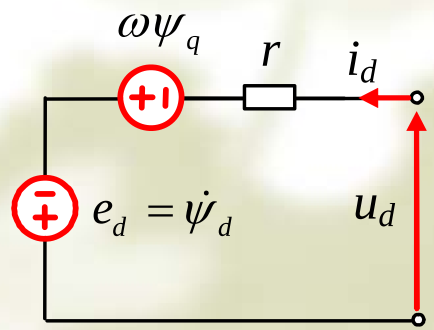




$$u_d = -\dot{\psi}_d - \omega\psi_q - ri_d$$




$$u_d = \dot{\psi}_d + \omega\psi_q - ri_d$$

# 3-6 同步电机的对称稳态运行-实用化基本方程

$$\begin{cases} u_d = -\dot{\psi}_d - \omega\psi_q - r\,i_d \\ u_q = -\dot{\psi}_q + \omega\psi_d - r\,i_q \\ u_f = \dot{\psi}_f + r_f i_f \\ 0 = \dot{\psi}_D + r_D i_D \\ 0 = \dot{\psi}_Q + r_Q i_Q \end{cases}$$

$$\begin{cases} \psi_d = x_d i_d + x_{ad} i_f + x_{ad} i_D \\ \psi_q = x_q i_q + x_{aq} i_Q \\ \psi_f = x_{ad} i_d + x_f i_f + x_{ad} i_D \\ \psi_D = x_{ad} i_d + x_{ad} i_f + x_D i_D \\ \psi_Q = x_{aq} i_q + x_Q i_Q \end{cases}$$

$u_d$=-$u_d$

$\omega$=1.0

$i_d$=-$i_d$

$$\begin{cases} u_d = \dot{\psi}_d + \psi_q - r\,i_d \\ u_q = -\dot{\psi}_q + \psi_d - r\,i_q \\ u_f = \dot{\psi}_f + r_f i_f \\ 0 = \dot{\psi}_D + r_D i_D \\ 0 = \dot{\psi}_Q + r_Q i_Q \end{cases}$$

$$\begin{cases} \psi_d = -x_d i_d + x_{ad} i_f + x_{ad} i_D \\ \psi_q = x_q i_q + x_{aq} i_Q \\ \psi_f = -x_{ad} i_d + x_f i_f + x_{ad} i_D \\ \psi_D = -x_{ad} i_d + x_{ad} i_f + x_D i_D \\ \psi_Q = x_{aq} i_q + x_Q i_Q \end{cases}$$

# 3-6 同步电机的对称稳态运行——电势方程

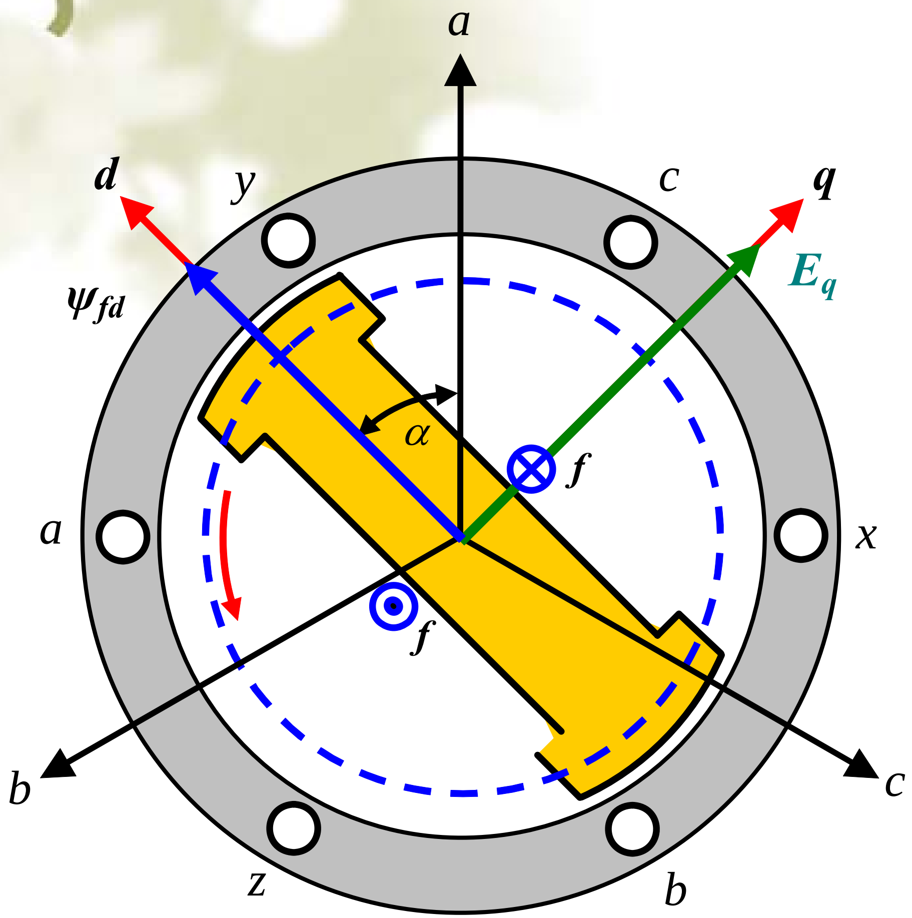
定子电流 $\begin{cases} i_d = -I\cos(\alpha-\theta) = -I\cos(\alpha_0-\theta_0) \\ i_q = I\sin(\alpha-\theta) = I\sin(\alpha_0-\theta_0) \end{cases}$

定子电压 $\begin{cases} u_d = x_q i_q \\ u_q = x_{ad} i_f - x_d i_d = \psi_{fd} - x_d i_d = E_q - x_d i_d \end{cases}$

$$\begin{cases} u_d = \dot{\psi}_d + \psi_q - r\, i_d \\ u_q = -\dot{\psi}_q + \psi_d - r\, i_q \end{cases} \xrightarrow{r=0} \begin{cases} u_d = \psi_q = x_q i_q \\ u_q = \psi_d = x_{ad} i_f - x_d i_d \end{cases}$$

$$\begin{cases} \psi_d = -x_d i_d + x_{ad} i_f + x_{ad} i_D \\ \psi_q = x_q i_q + x_{aq} i_Q \end{cases} \Rightarrow \begin{cases} \psi_d = -x_d i_d + x_{ad} i_f \\ \psi_q = x_q i_q \end{cases}$$

# 3-6 同步电机的对称稳态运行——空载电势的物理意义

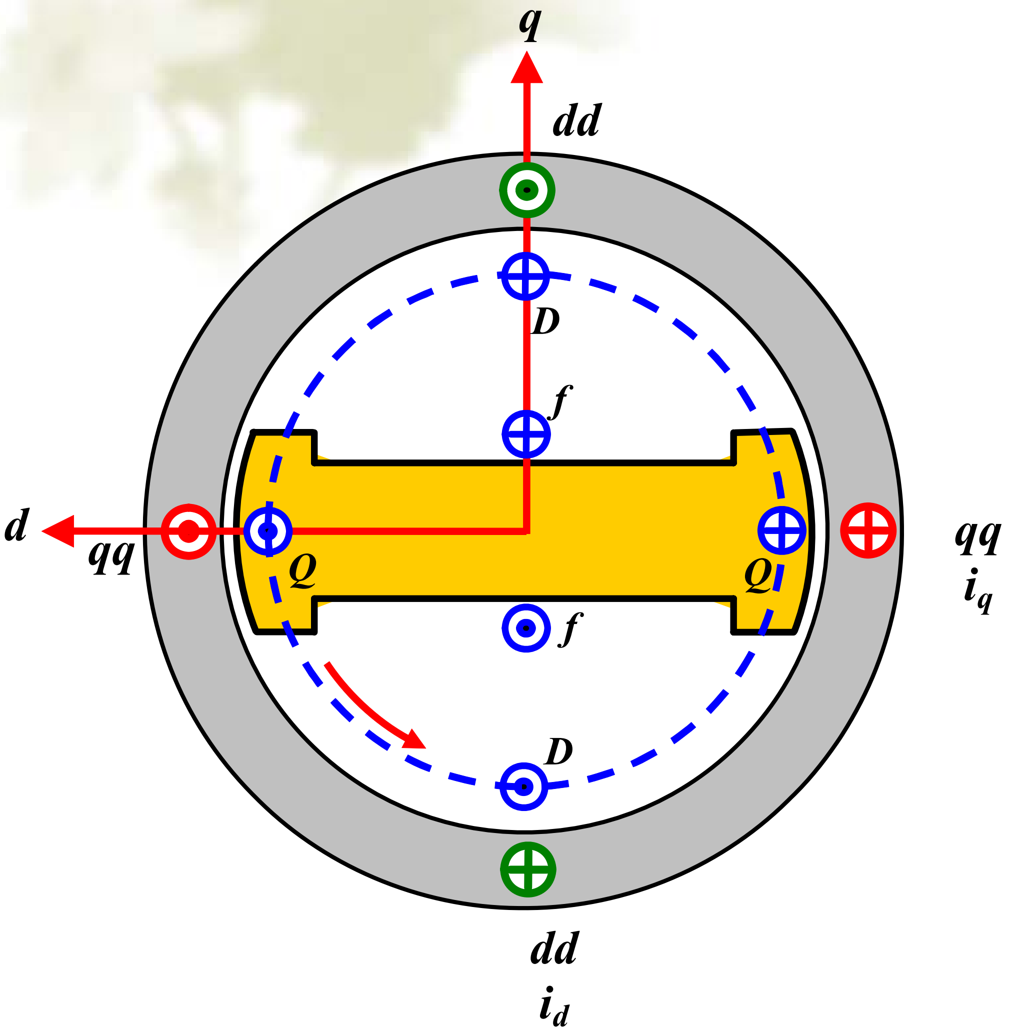


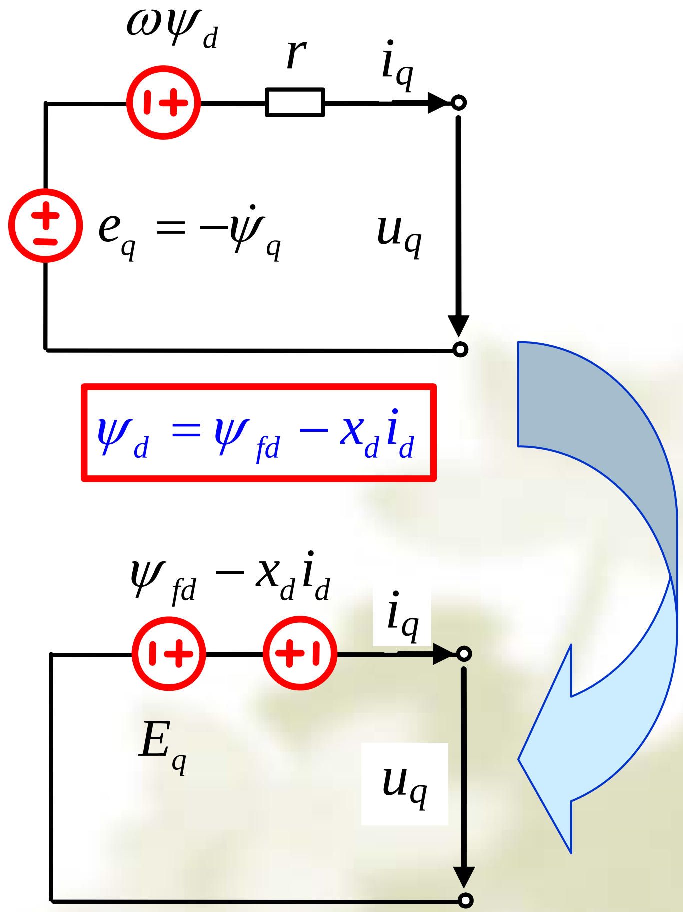

空载电势

$$E_q = \psi_{fd} = x_{ad} i_f$$

$$\begin{cases} e_a = E_q \sin\alpha \\ e_b = E_q \sin(\alpha - 120^\circ) \\ e_c = E_q \sin(\alpha + 120^\circ) \end{cases}$$

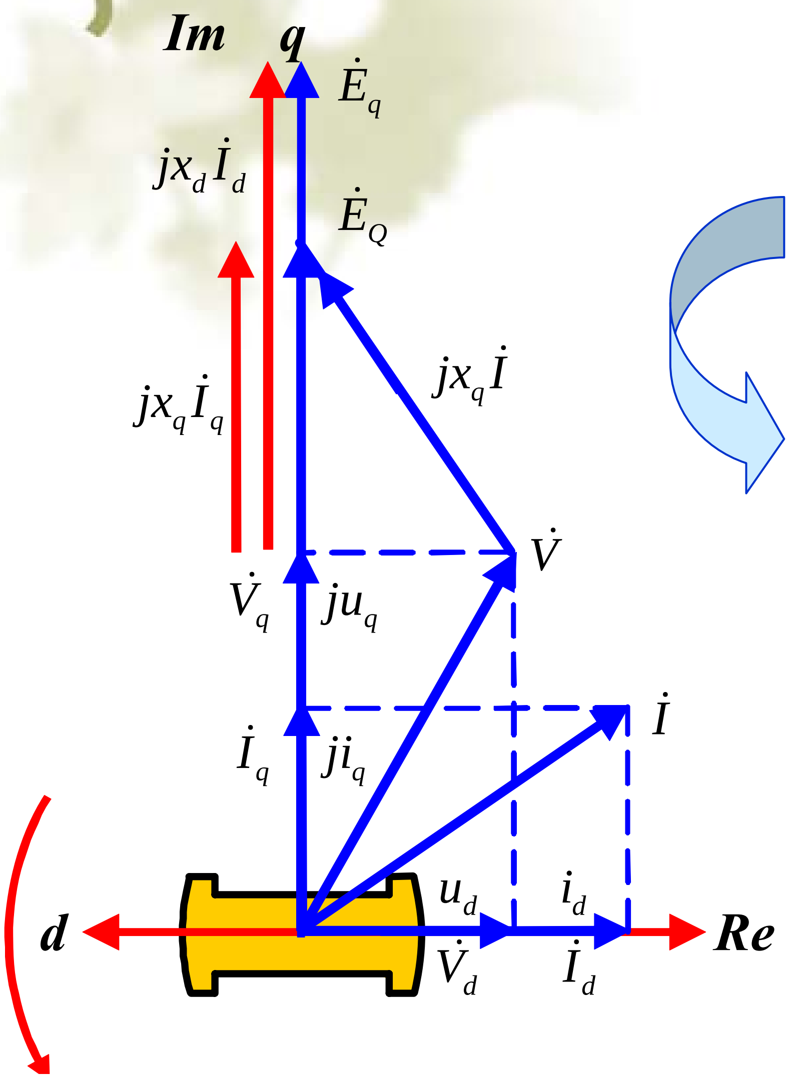
# 3-6 同步电机的对称稳态运行——空载电势的物理意义




$$\psi_d = \psi_{fd} - x_d i_d$$

# 3-6 同步电机的对称稳态运行——相量图




$$\begin{cases} u_d = x_q i_q \\ u_q = E_q - x_d i_d \end{cases}$$

$$\dot{E}_q = jE_q, \dot{V}_d = u_d, \dot{I}_d = i_d$$
$$\dot{I}_q = ji_q, \dot{V}_q = ju_q$$

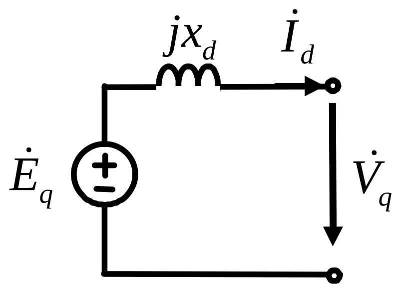
$$\begin{cases} u_d = -jx_q \times ji_q \\ ju_q = jE_q - jx_d i_d \end{cases}$$

$$\begin{cases} \dot{V}_d = -jx_q \dot{I}_q \\ \dot{V}_q = \dot{E}_q - jx_d \dot{I}_d \end{cases}$$

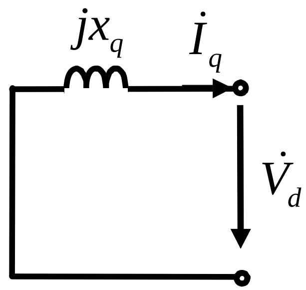
$$\dot{V} = \dot{E}_q - jx_d \dot{I}_d - jx_q \dot{I}_q = \dot{E}_Q - jx_q \dot{I}$$
$$\dot{E}_Q = \dot{E}_q - j(x_d - x_q)\dot{I}_d$$

# 3-6 同步电机的对称稳态运行——等值电路

$$\begin{cases} \dot{V}_d = -jx_q\dot{I}_q \\ \dot{V}_q = \dot{E}_q - jx_d\dot{I}_d \end{cases}$$



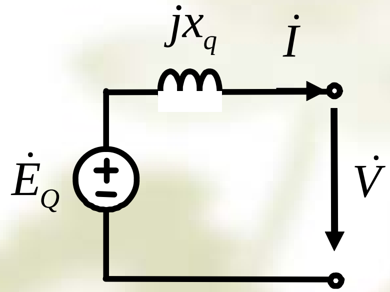
纵轴向等值电路



横轴向等值电路

$$\dot{V} = \dot{E}_q - jx_d\dot{I}_d - jx_q\dot{I}_q = \dot{E}_Q - jx_q\dot{I}$$

$$\dot{E}_Q = \dot{E}_q - j(x_d - x_q)\dot{I}_d$$



等值隐极机电路

等值隐极机电势：$\dot{E}_Q = \dot{E}_q - j(x_d - x_q)\dot{I}_d$

# 本章小结

## 一、同步电机原始方程式

1. 电压方程式和磁链方程式：注意正方向的定义；

2. 磁链方程式中，各绕组及相互之间电感系数随转子运动发生变化；使得同步电机电压方程式为时变系数微分方程式；难以分析求解；

3. 各绕组及相互之间电感系数时变的原因是定转子相对运动和转子不对称造成磁通经过的磁路磁阻变化—磁路分析方法；

# 本章小结

## 二、Park变换：abc坐标系—dq0坐标系

1. Park变换：将定子abc三相绕组等效为假想绕组dd和qq绕组，dd和qq分别位于转子纵轴和横轴向；

2. 同步电机纵轴向三个绕组：定子等效绕组dd，励磁绕组f和纵轴阻尼绕组D，相互之间位置不变，互感磁通沿纵轴向构成磁路，电感系数为常数；

3. 横轴向两个绕组：定子等效绕组qq，横轴阻尼绕组Q，相互之间位置不变，互感磁通沿纵轴向构成磁路，电感系数为常数；

4. 纵轴和交轴向绕组成90度，无互感耦合磁通，相互间互感系数为零；

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# 本章小结

5. Park变换—对定子侧电压电流等效变换；abc坐标系中定子侧各种频率的电压电流和磁链与dq0坐标系中各频率分量的对应关系；

## 三、同步电机基本方程及实用化（标幺值形式）

1. 采用伪静止线组可以合理解释电势方程;

2. 同步电机的电抗：基于公共互感磁通的假设；漏电抗，纵轴同步反应电抗；交轴同步反应电抗；

3. 同步电机对称稳态分析：相量方程式，等值电路，相量图。$E_q$、$E_Q$的概念

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# 《电力系统分析》复习思考题

1. 什么是理想化同步电机?

2. 同步电机定子绕组和转子绕组的自感系数和互感系数哪些同转子的位置角有关?它们有怎样的变化规律?

3. 导致同步电机电感系数随转子位置角变化的原因是什么?

4. 在同步发电机的稳态运行分析中怎样处理电枢反应的?这样处理有什么好处?

# 《电力系统分析》复习思考题

5. Dq0坐标系下的直流分量和基频分量分别与abc坐标系的什么分量对应？为什么？

6. 不对称三相正弦变量进行dq0坐标变换会得到什么结果？如果(1)三相量中含零序分量，(2)三相量中含负序分量，则又有什么结果？

7. Dq0坐标系的电势方程中，定子电势由哪些分量组成?这些分量是怎样产生的？

8. dq0坐标系的磁链方程中，是什么原因使定子、转子绕组之间的互感系数变得不具有互易性？可以用什么方法来解决这个问题？

9. 实际应用同步电机基本方程式时，常采用哪些简化假设？这些假设忽略了什么因素？给计算带来哪些方便？其适用范围怎样？

10. 什么是实用正向？为什么要对同步电机基本方程式中部分变量的假定正方向进行调整？

11. 试作出凸极发电机稳态运行时的电势相量图。

12. 什么叫假想电势EQ？它怎样计算？有什么用处？

# 习 题

## Ex 3-6，3-7，3-8

建议考虑其他题

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# To Be Continued