

华中科技大学

Huazhong University of  
Science and Technology

2009-2010学年度第一学期

2009.11.08—2010.01.30



《电力系统分析》 (I)

主讲教师：孙海顺

E-mail: [haishunsun@hust.edu.cn](mailto:haishunsun@hust.edu.cn)

## 第四章 电力网络的数学模型

4-1 节点导纳矩阵

4-2 网络方程的解法

4-3 节点阻抗矩阵

4-4 节点编号顺序的优化

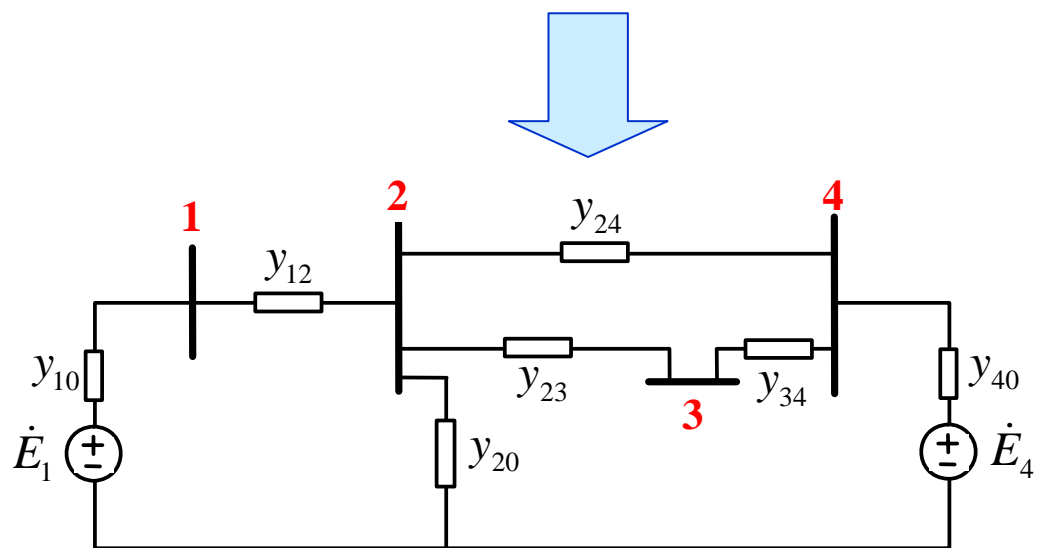
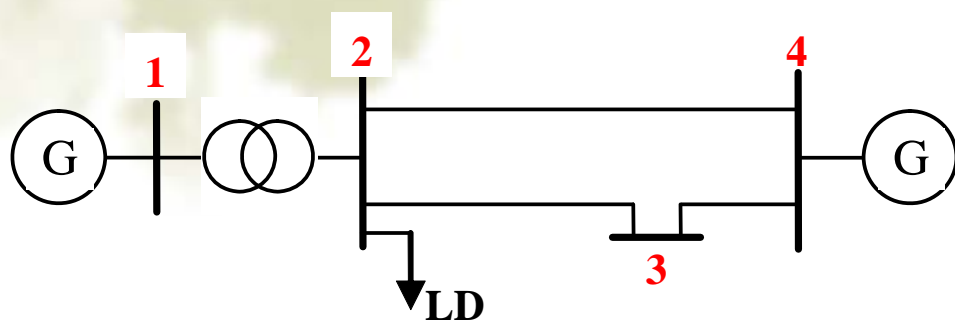
# 第四章 电力网络的数学模型

## 4-1 节点导纳矩阵

1. 用于短路计算的电力系统数学模型
2. 节点方程
3. 节点导纳矩阵元素的物理意义
4. Y阵的修改（网络结构变化、故障等）

## 4-1 节点导纳矩阵

### 1. 用于短路计算的电力系统数学模型

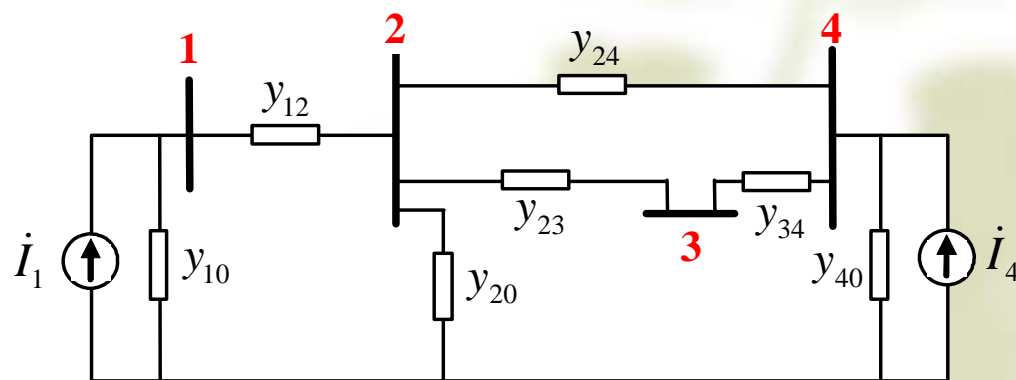
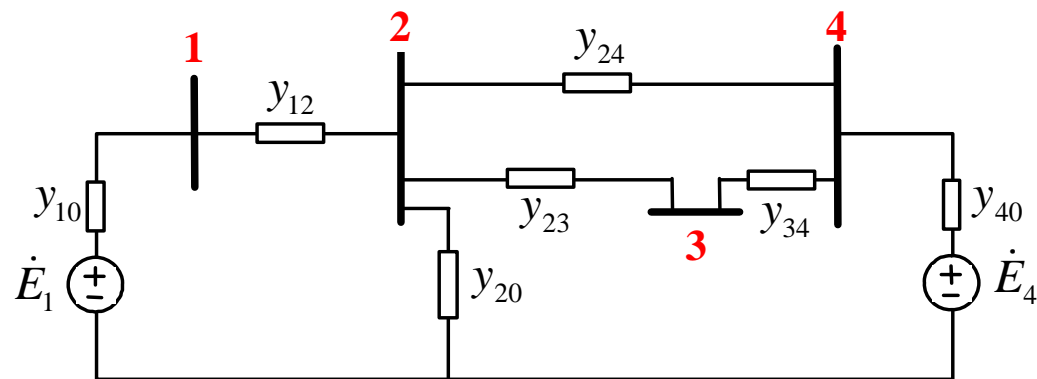


- ❖ 发电机：电势源支路
- ❖ 电力网络：一相等值电路，略去变压器励磁支路和线路电容
- ❖ 负荷：恒定阻抗
- ❖ 零电位参考节点不予编号

## 4-1 节点导纳矩阵

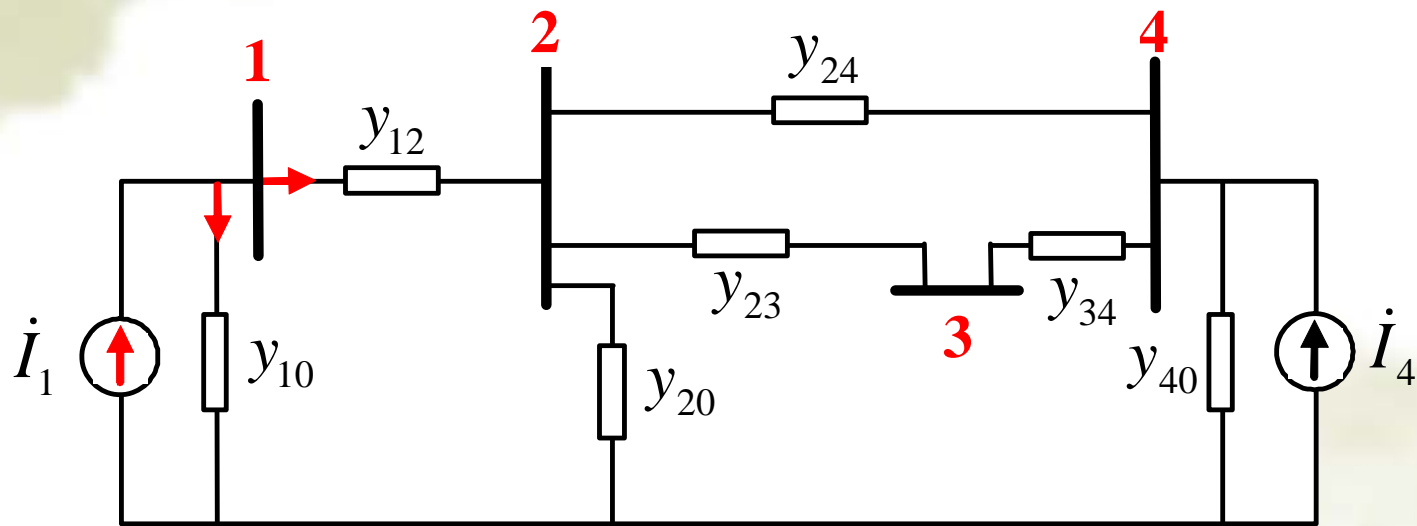
### 2. 节点方程

电势源支路转换为  
电流源支路，  
根据KCL定律列  
写每一个节点的  
电流方程式。



# 4-1 节点导纳矩阵

## 2. 节点方程—节点1

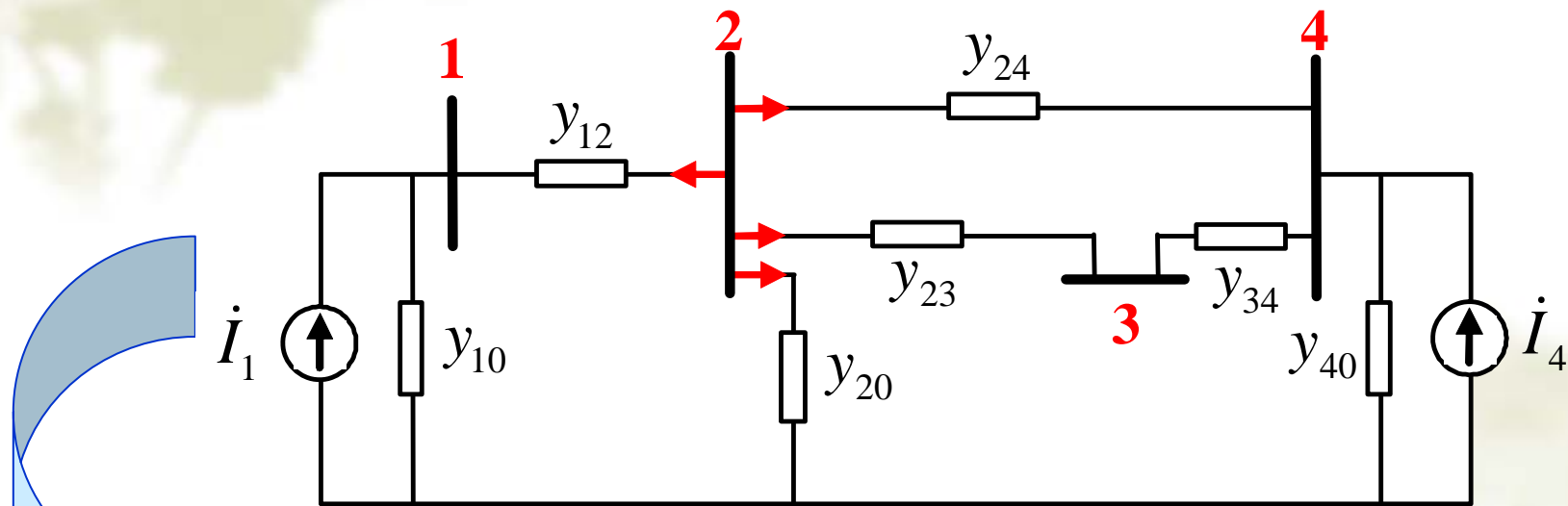


$$\dot{I}_1 = y_{10}\dot{V}_1 + y_{12}(\dot{V}_1 - \dot{V}_2)$$

$$\dot{I}_1 = (y_{10} + y_{12})\dot{V}_1 - y_{12}\dot{V}_2$$

# 4-1 节点导纳矩阵

## 2. 节点方程—节点2

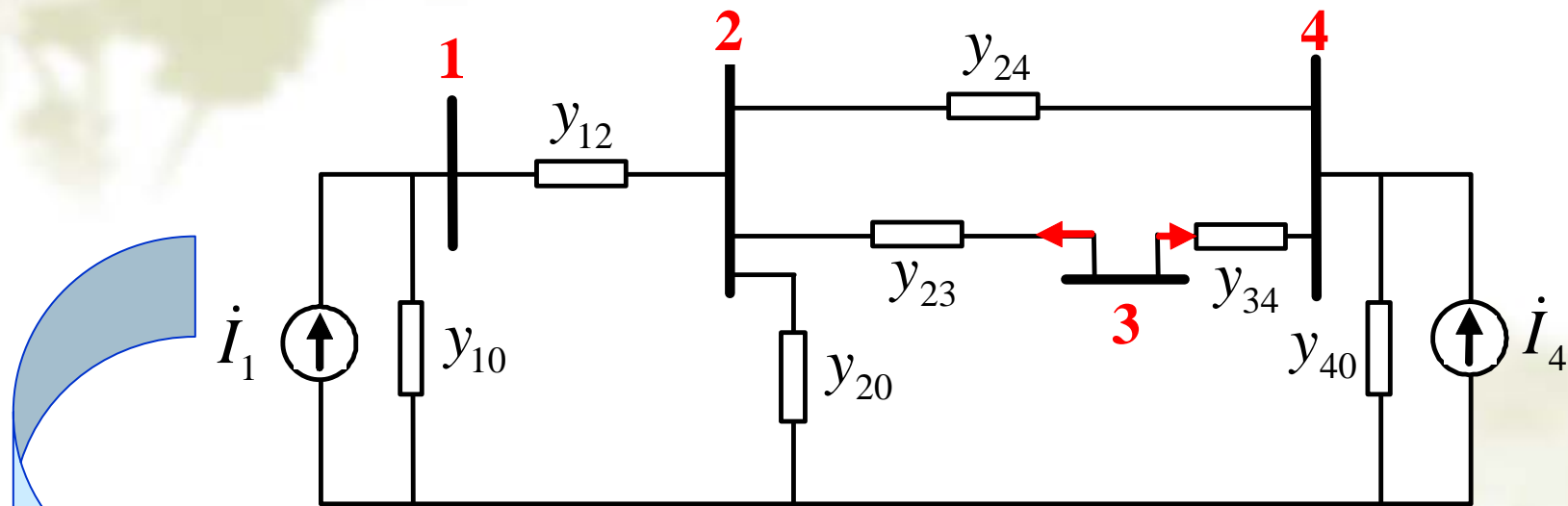


$$0 = y_{12}(\dot{V}_2 - \dot{V}_1) + y_{20}\dot{V}_2 + y_{23}(\dot{V}_2 - \dot{V}_3) + y_{24}(\dot{V}_2 - \dot{V}_4)$$

$$0 = -y_{12}\dot{V}_1 + (y_{12} + y_{20} + y_{23} + y_{24})\dot{V}_2 - y_{23}\dot{V}_3 - y_{24}\dot{V}_4$$

# 4-1 节点导纳矩阵

## 2. 节点方程—节点3



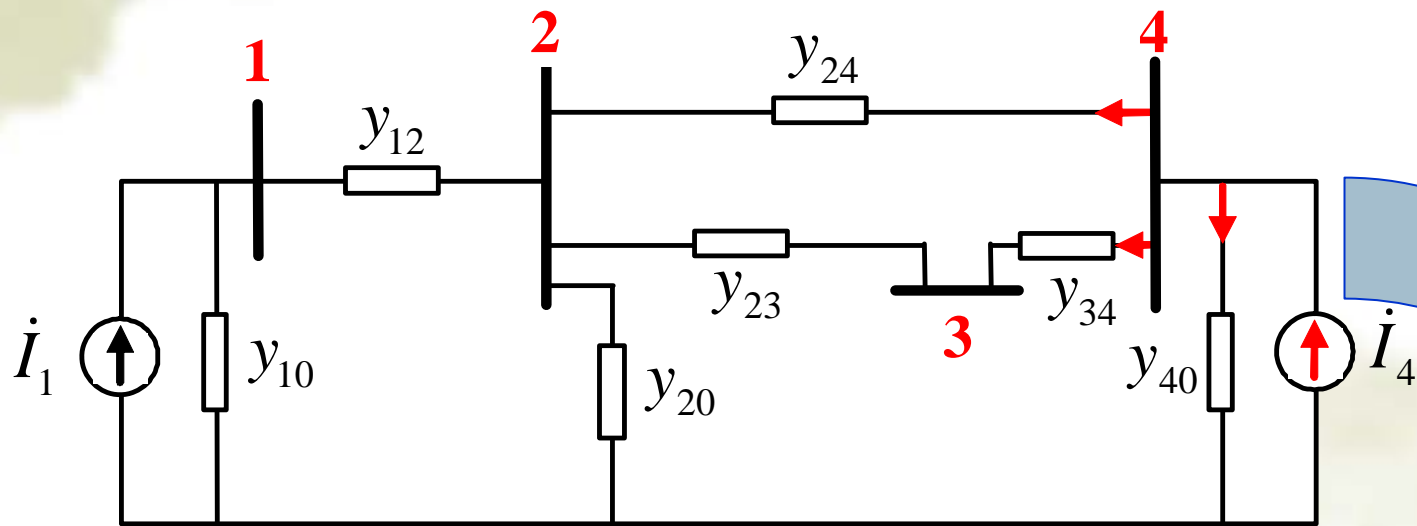
$$0 = y_{23}(\dot{V}_3 - \dot{V}_2) + y_{34}(\dot{V}_3 - \dot{V}_4)$$

$$0 = -y_{23}\dot{V}_2 + (y_{23} + y_{34})\dot{V}_3 - y_{34}\dot{V}_4$$



# 4-1 节点导纳矩阵

## 2. 节点方程—节点4



$$\dot{I}_4 = y_{24}(\dot{V}_4 - \dot{V}_2) + y_{34}(\dot{V}_4 - \dot{V}_3) + y_{40}\dot{V}_4$$

$$\dot{I}_4 = -y_{24}\dot{V}_2 - y_{34}\dot{V}_3 + (y_{24} + y_{34} + y_{40})\dot{V}_4$$

## 4-1 节点导纳矩阵

### 2. 节点方程的矩阵形式

$$\begin{cases} \dot{I}_1 = (y_{10} + y_{12})\dot{V}_1 - y_{12}\dot{V}_2 \\ 0 = -y_{12}\dot{V}_1 + (y_{12} + y_{20} + y_{23} + y_{24})\dot{V}_2 - y_{23}\dot{V}_3 - y_{24}\dot{V}_4 \\ 0 = -y_{23}\dot{V}_3 + (y_{23} + y_{34})\dot{V}_3 - y_{34}\dot{V}_4 \\ \dot{I}_4 = -y_{24}\dot{V}_2 - y_{34}\dot{V}_3 + (y_{24} + y_{34} + y_{40})\dot{V}_4 \end{cases}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{24} = Y_{42} = -y_{24}$$

$$Y_{34} = Y_{43} = -y_{34}$$

$$\begin{bmatrix} \dot{I}_1 \\ 0 \\ 0 \\ \dot{I}_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 0 & Y_{32} & Y_{33} & Y_{34} \\ 0 & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}$$

$$Y_{11} = y_{10} + y_{12}$$

$$Y_{22} = y_{12} + y_{20} + y_{23} + y_{24}$$

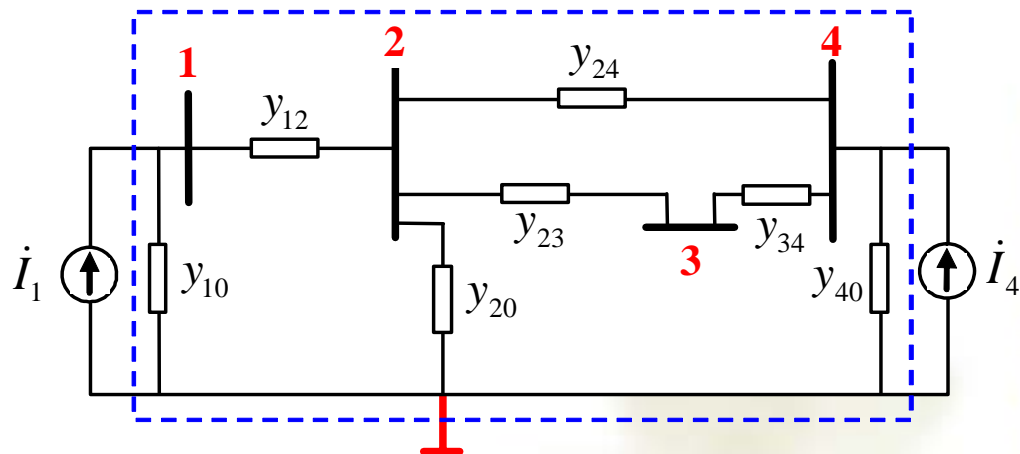
$$Y_{33} = y_{23} + y_{34}$$

$$Y_{44} = y_{24} + y_{34} + y_{40}$$

# 4-1 节点导纳矩阵

## 2. 节点方程的矩阵形式

$$\begin{bmatrix} \dot{I}_1 \\ 0 \\ 0 \\ \dot{I}_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 0 & Y_{32} & Y_{33} & Y_{34} \\ 0 & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}$$



$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}$$

节点注入电流:  $\dot{\mathbf{I}} = [\dot{I}_1 \quad \dot{I}_2 \quad \dot{I}_3 \quad \dot{I}_4]^T$

节点电压:  $\dot{\mathbf{V}} = [\dot{V}_1 \quad \dot{V}_2 \quad \dot{V}_3 \quad \dot{V}_4]^T$

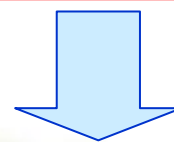
$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$

## 4-1 节点导纳矩阵

### 2. 节点方程— $n$ 个独立节点的电力网络数学模型

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix}$$

线性代数方程



$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

$Y_{ii}$

节点 $i$ 自导纳

$Y_{ij}$

节点 $ij$ 间互导纳

节点导纳矩阵

$\mathbf{Y}$

## 4-1 节点导纳矩阵

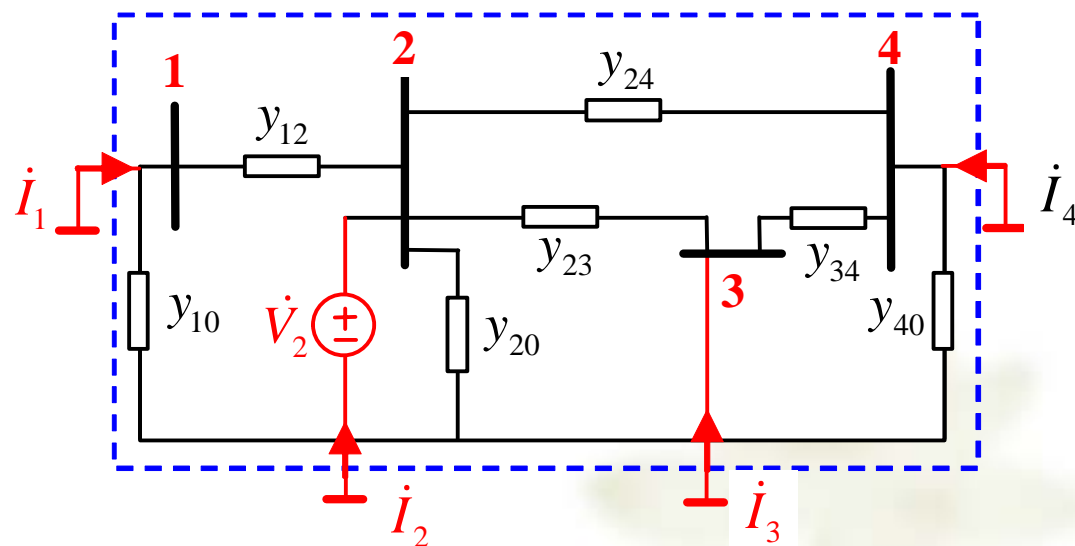
### 3. 节点导纳矩阵元素的物理意义

$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

$$\begin{cases} \dot{V}_i \neq 0, \\ \dot{V}_j = 0, \\ (j \neq i) \end{cases}$$

$$\begin{cases} \dot{I}_i = \sum_{j=1}^n Y_{ij} \dot{V}_j \\ \dot{I}_j = \sum_{k=1}^n Y_{jk} \dot{V}_k \end{cases}$$

$$\begin{aligned} \dot{I}_i &= Y_{ii} \dot{V}_i \Rightarrow Y_{ii} = \dot{I}_i / \dot{V}_i \\ \dot{I}_j &= Y_{ji} \dot{V}_i \Rightarrow Y_{ji} = \dot{I}_j / \dot{V}_i \end{aligned}$$



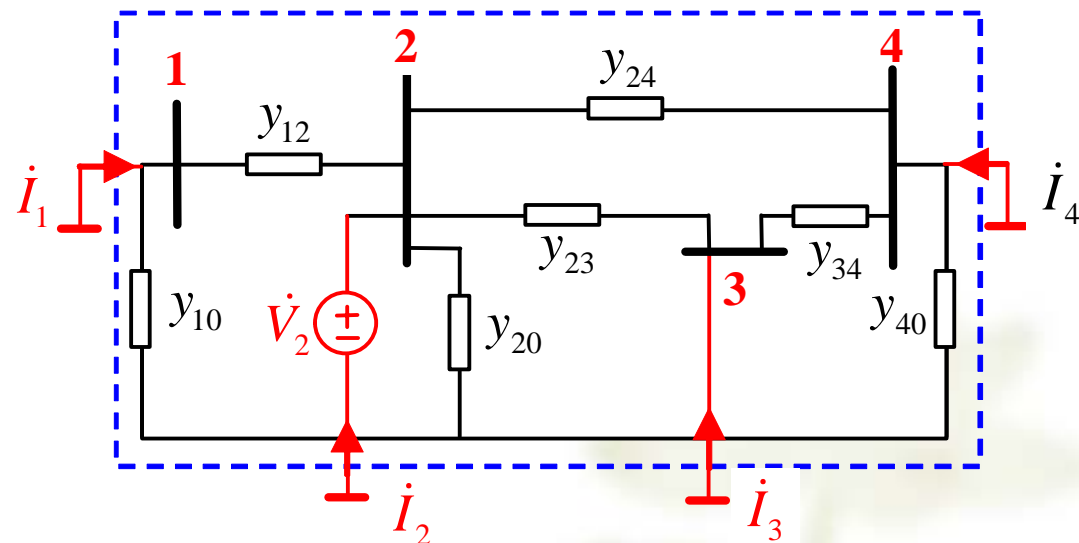
$$\begin{aligned} Y_{22} &= \dot{I}_2 / \dot{V}_2 = y_{20} + y_{12} + y_{23} + y_{24} \\ Y_{12} &= \dot{I}_1 / \dot{V}_2 = -y_{12}, \quad Y_{32} = \dot{I}_3 / \dot{V}_2 = -y_{23} \\ Y_{42} &= \dot{I}_4 / \dot{V}_2 = -y_{24} \end{aligned}$$

## 4-1 节点导纳矩阵

### 3. 节点导纳矩阵元素的物理意义——自导纳

$Y_{ii}$ : 当网络中除节点*i*以外所有节点都接地时, 从节点*i*注入网络的电流同施加于节点*i*的电压之比

$Y_{ii}$ : 与节点*i*相连的所有支路导纳之和

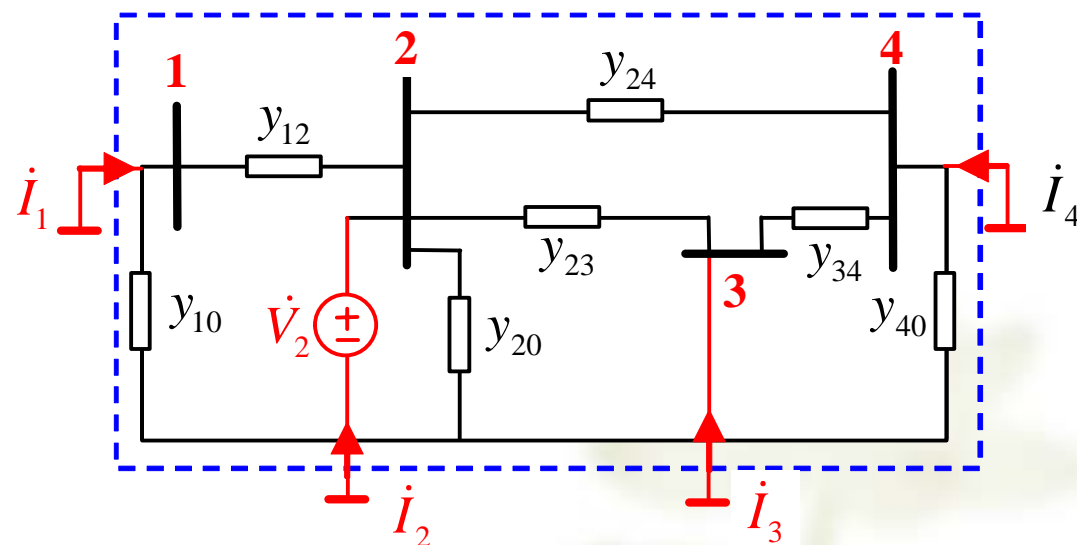


$$Y_{22} = \dot{I}_2 / \dot{V}_2 = y_{20} + y_{12} + y_{23} + y_{24}$$

## 4-1 节点导纳矩阵

### 3. 节点导纳矩阵元素的物理意义——互导纳

$Y_{ji}$ : 当网络中除节点  $i$  以外所有节点都接地时, 从节点  $j$  注入网络的电流同施加于节点  $i$  的电压之比



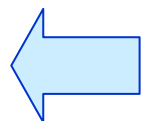
节点  $j$  的电流实际上是自网络流出并进入地中的电流, 所以  $Y_{jk}$  应等于节点  $j$ 、 $i$  之间连接支路导纳的负值

$$Y_{12} = I_1 / \dot{V}_2 = -y_{12}$$
$$Y_{32} = I_3 / \dot{V}_2 = -y_{23}$$
$$Y_{42} = I_4 / \dot{V}_2 = -y_{24}$$

## 4-1 节点导纳矩阵

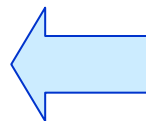
### 3. 节点导纳矩阵的特点

❖ 直观易求



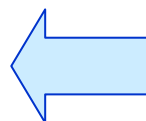
$$Y_{ii} = y_{i0} + \sum y_{ij}, \quad Y_{ij} = -\sum y_{ij}$$

❖ 对称矩阵



$$Y_{ij} = Y_{ji} = -\sum y_{ij}$$

❖ 稀疏矩阵



如果 $ij$ 之间没有直接支路连接, 则  
 $Y_{ij} = Y_{ji} = 0$

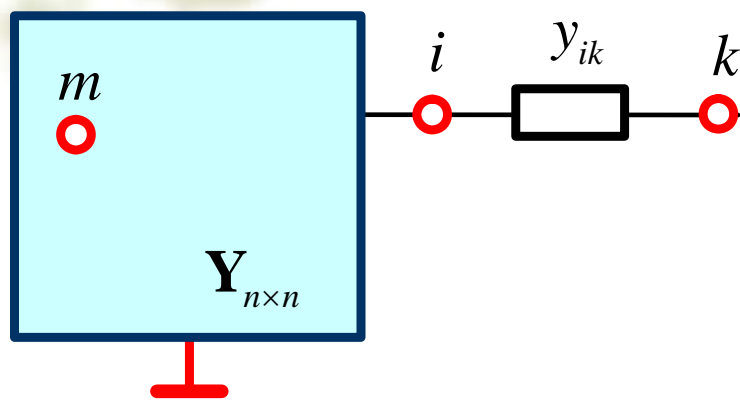
$$Y_{n \times n}$$

❖ 设每个节点平均有**10**条出线, 则节点导纳矩阵非零元素为**11n**个, 稀疏度**11/n**



## 4-1 节点导纳矩阵

### 4. Y阵的修改——增加树枝



$$\begin{bmatrix}
 Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1n} & 0 \\
 \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\
 Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{in} & Y_{ik} \\
 \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\
 Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nn} & 0 \\
 \hline
 0 & \cdots & Y_{ki} & \cdots & 0 & Y_{kk}
 \end{bmatrix}$$

$$Y_{ii} = Y_{ii}^{(0)} + y_{ik}$$

$$Y_{kk} = y_{ik}$$

$$Y_{ik} = Y_{ki} = -y_{ik}$$

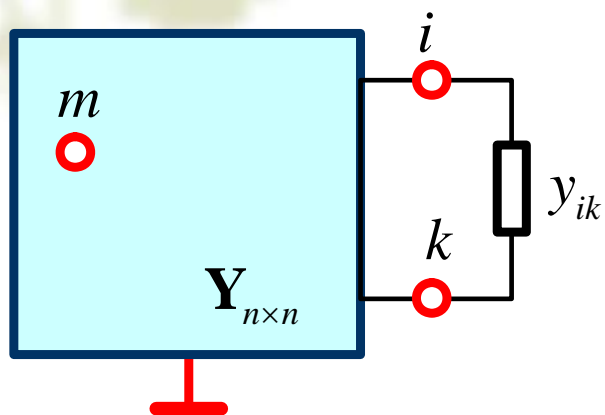
$$Y_{mk} = Y_{km} = 0$$

$$Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

Y阵增加一行一列

## 4-1 节点导纳矩阵

### 4. Y阵的修改——增加连支



$$\begin{bmatrix}
 Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1k} & \cdots & Y_{1n} \\
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{ik} & \cdots & Y_{in} \\
 \hline
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{k1} & \cdots & Y_{ki} & \cdots & Y_{kk} & \cdots & Y_{kn} \\
 \hline
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nk} & \cdots & Y_{nn}
 \end{bmatrix}$$

Y阵阶数保持不变

$$Y_{ii} = Y_{ii}^{(0)} + y_{ik}$$

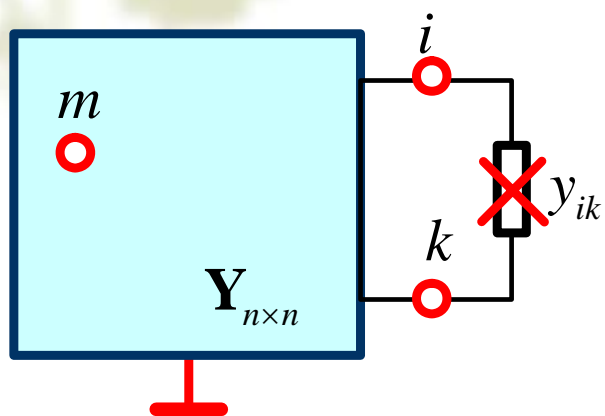
$$Y_{kk} = Y_{kk}^{(0)} + y_{ik}$$

$$Y_{ik} = Y_{ki} = Y_{ik}^{(0)} - y_{ik}$$

$$Y_{mk} = Y_{km} = Y_{mk}^{(0)}, \quad Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

## 4-1 节点导纳矩阵

### 4. Y阵的修改——删除连支



$$\begin{bmatrix}
 Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1k} & \cdots & Y_{1n} \\
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{ik} & \cdots & Y_{in} \\
 \hline
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{k1} & \cdots & Y_{ki} & \cdots & Y_{kk} & \cdots & Y_{kn} \\
 \hline
 \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nk} & \cdots & Y_{nn}
 \end{bmatrix}$$

$$Y_{ii} = Y_{ii}^{(0)} - y_{ik}$$

$$Y_{kk} = Y_{kk}^{(0)} - y_{ik}$$

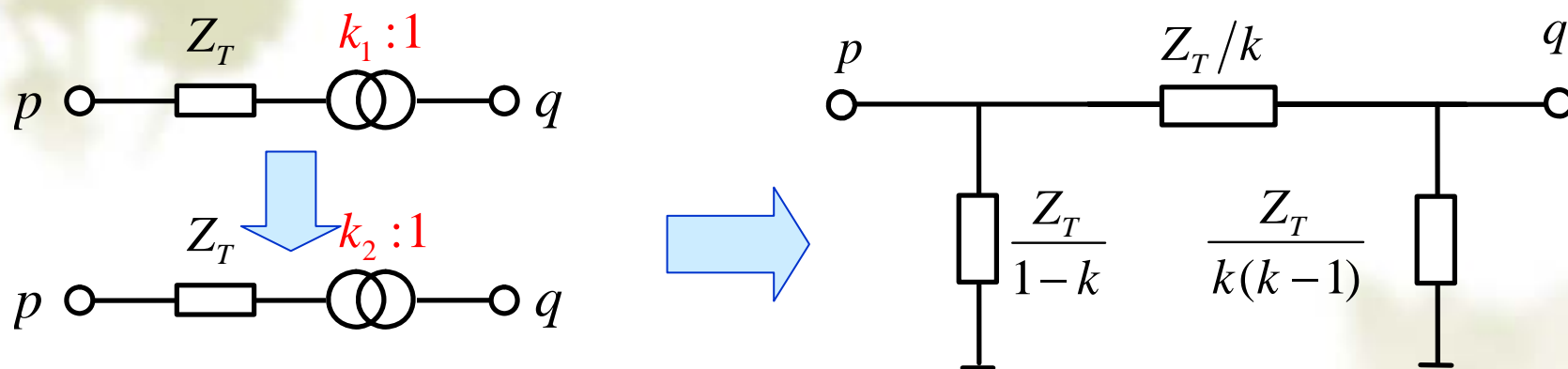
$$Y_{ik} = Y_{ki} = Y_{ik}^{(0)} + y_{ik}$$

Y阵阶数保持不变

$$Y_{mk} = Y_{km} = Y_{mk}^{(0)}, \quad Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

## 4-1 节点导纳矩阵

### 4. Y阵的修改——改变变压器变比



$$Y_{pp} = Y_{pp}^{(0)} - \Delta Y_{pp}^{(1)} + \Delta Y_{pp}^{(2)}$$

$$Y_{qq} = Y_{qq}^{(0)} - \Delta Y_{qq}^{(1)} + \Delta Y_{qq}^{(2)}$$

$$Y_{pq} = Y_{qp} = Y_{pq}^{(0)} - \Delta Y_{pq}^{(1)} + \Delta Y_{pq}^{(2)}$$

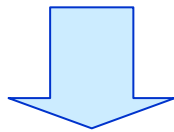
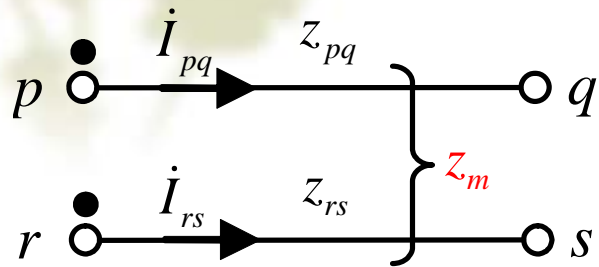
$$\Delta Y_{pp} = k/Z_T + (1-k)/Z_T = 1/Z_T$$

$$\Delta Y_{qq} = k/Z_T + k(k-1)/Z_T = k^2/Z_T$$

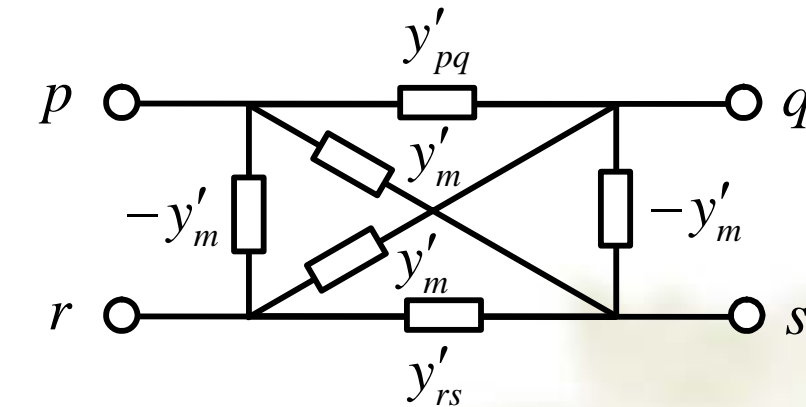
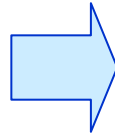
$$\Delta Y_{pq} = \Delta Y_{qp} = -k/Z_T$$

## 4-1 节点导纳矩阵

### 4. Y阵的修改—支路间存在互感



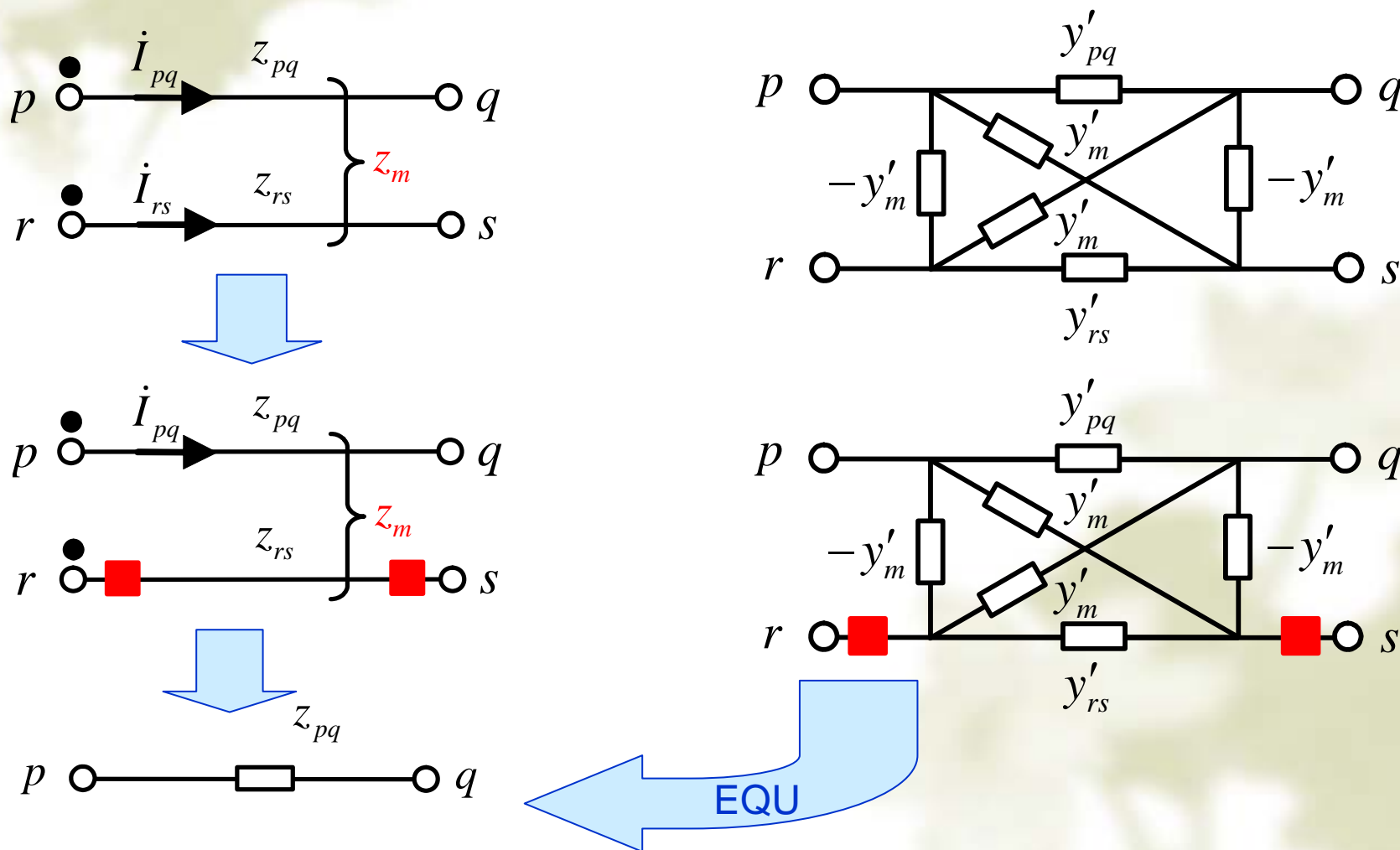
$$\begin{bmatrix} \dot{V}_p & -\dot{V}_q \\ \dot{V}_r & -\dot{V}_s \end{bmatrix} = \begin{bmatrix} z_{pq} & z_m \\ z_m & z_{rs} \end{bmatrix} \begin{bmatrix} \dot{I}_{pq} \\ \dot{I}_{rs} \end{bmatrix}$$



$$\begin{bmatrix} \dot{I}_{pq} \\ \dot{I}_{rs} \end{bmatrix} = \begin{bmatrix} y'_{pq} & y'_m \\ y'_m & y'_{rs} \end{bmatrix} \begin{bmatrix} \dot{V}_p & -\dot{V}_q \\ \dot{V}_r & -\dot{V}_s \end{bmatrix}$$

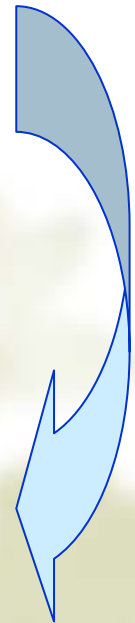
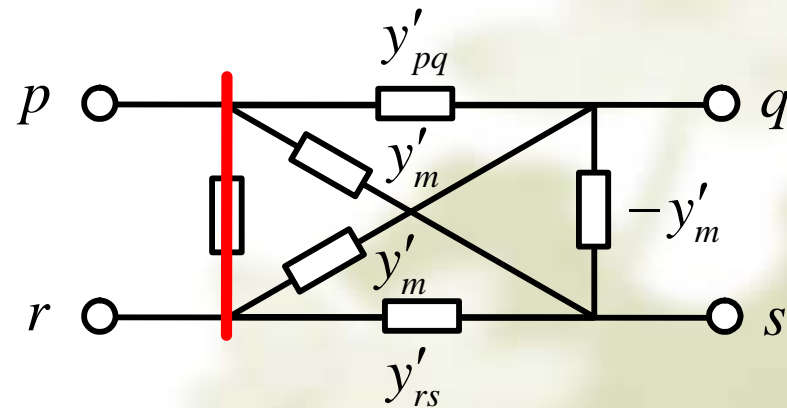
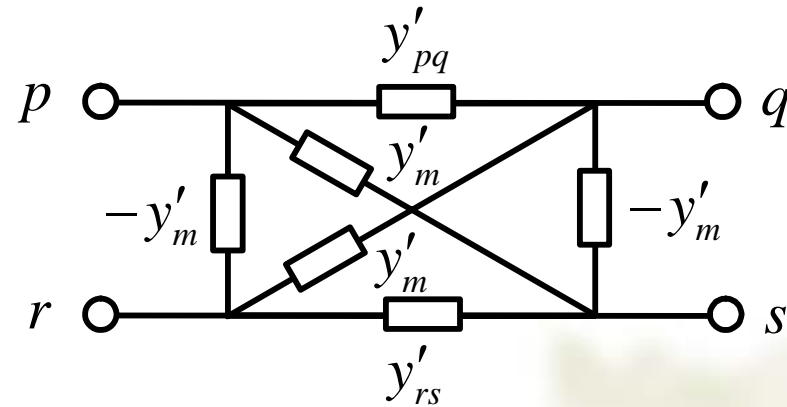
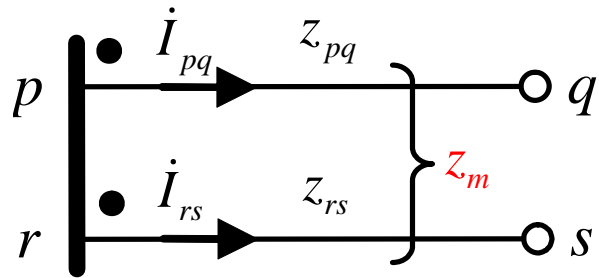
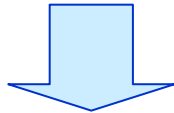
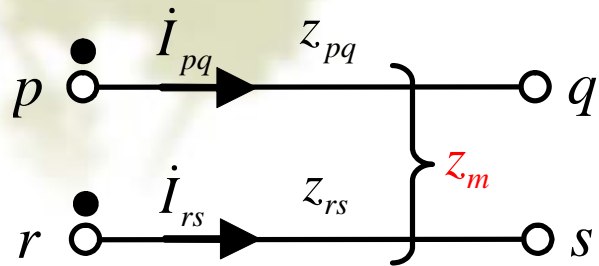
# 4-1 节点导纳矩阵

## 4. Y阵的修改——断开互感支路



# 4-1 节点导纳矩阵

## 4. Y阵的修改——一端互联的互感支路



## 4-3 节点阻抗矩阵

### 1. 节点阻抗矩阵

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix}$$

$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

$$\dot{\mathbf{V}} = \mathbf{Z}\dot{\mathbf{I}}$$

$$Z_{ii}$$

节点*i*自阻抗

$$Z_{ij}$$

节点*ij*间互阻抗

节点阻抗矩阵

$$\mathbf{Z} = \mathbf{Y}^{-1}$$



## 4-3 节点阻抗矩阵

### 2. 节点阻抗矩阵元素的物理意义

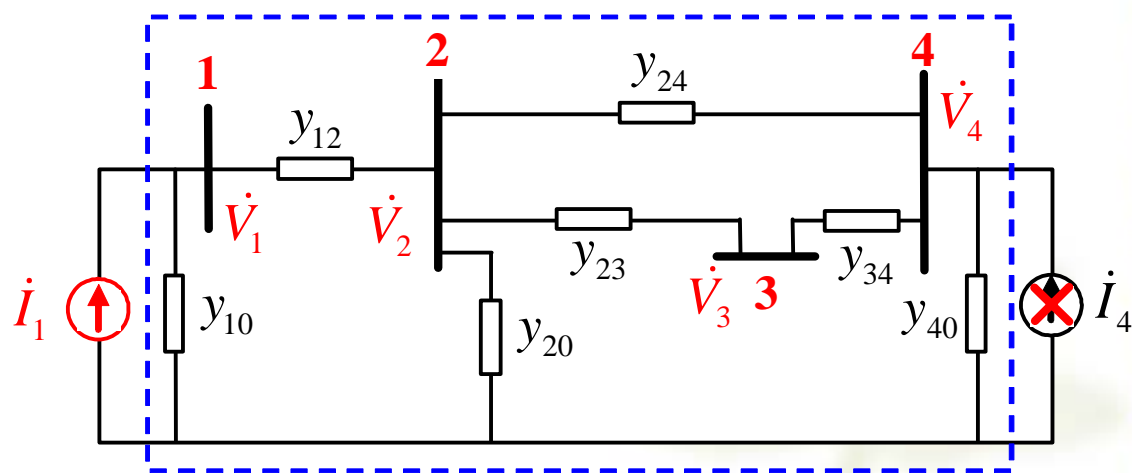
$$\dot{\mathbf{V}} = \mathbf{Z}\dot{\mathbf{I}}$$

$$\dot{I}_i \neq 0,$$

$$\dot{I}_j = 0,$$

$$(j \neq i)$$

$$\begin{cases} \dot{V}_i = \sum_{j=1}^n Z_{ij} \dot{I}_j \\ \dot{V}_j = \sum_{k=1}^n Z_{jk} \dot{I}_k \end{cases}$$



$$\dot{V}_i = Z_{ii} \dot{I}_i \Rightarrow Z_{ii} = \dot{V}_i / \dot{I}_i$$

$$\dot{V}_j = Z_{ji} \dot{I}_i \Rightarrow Z_{ji} = \dot{V}_j / \dot{I}_i$$

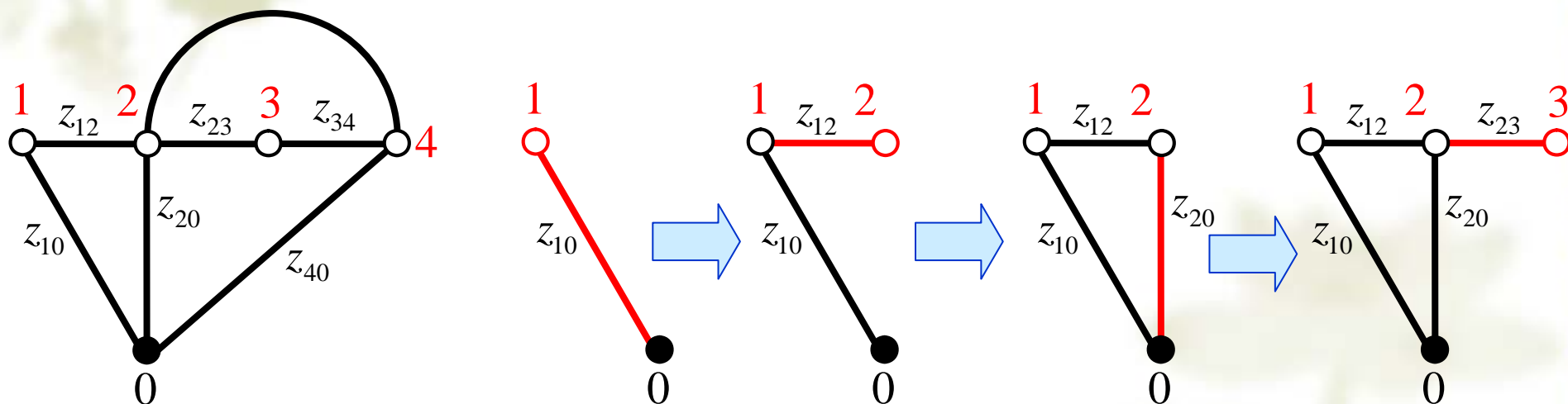
❖ 对称矩阵

❖ 满阵

❖ 计算复杂

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-基本过程



$$Z_{4 \times 4}$$

$$Z_{1 \times 1}$$

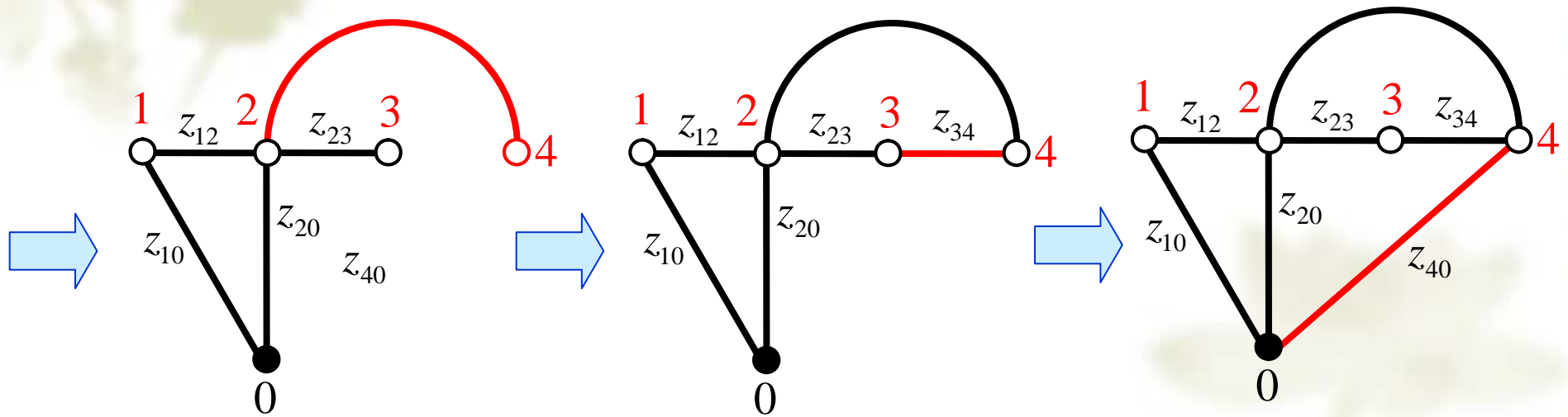
$$Z_{2 \times 2}$$

$$Z'_{2 \times 2}$$

$$Z_{3 \times 3}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-基本过程



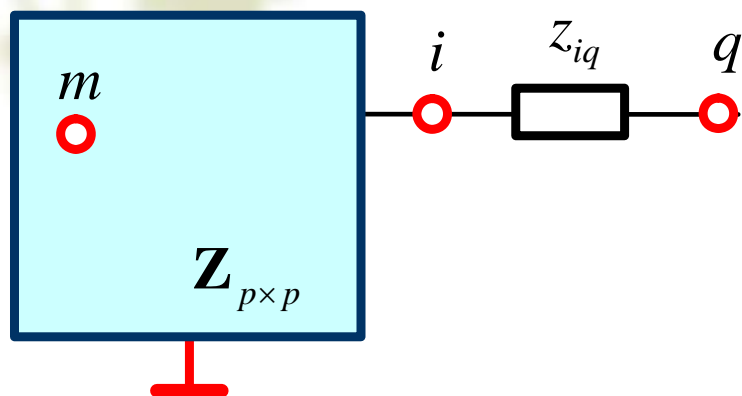
$$\mathbf{Z}'_{4 \times 4}$$

$$\mathbf{Z}''_{4 \times 4}$$

$$\mathbf{Z}_{4 \times 4}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加树枝



$$\mathbf{Z}_{p \times p}$$

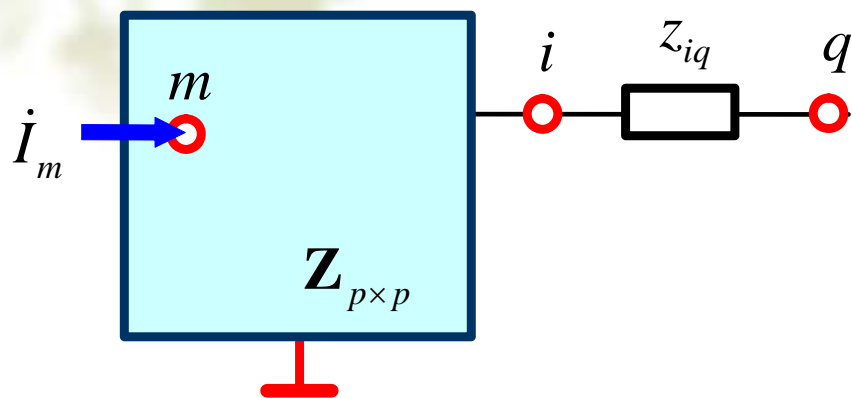
$$\mathbf{Z}_{(p+1) \times (p+1)}$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

Z阵增加一行一列

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加树枝

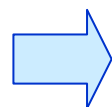


$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

$m$ 节点单独注入电流 $\dot{I}_m$

$$\dot{V}_i = Z_{im} \dot{I}_m$$

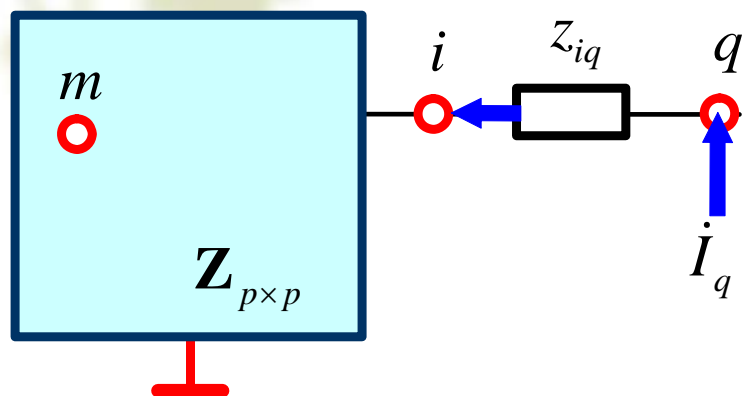
$$\dot{V}_q = Z_{qm} \dot{I}_m = \dot{V}_i = Z_{im} \dot{I}_m$$



$$Z_{qm} = Z_{im}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加树枝



$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

$q$ 节点单独注入电流 $\dot{I}_q$

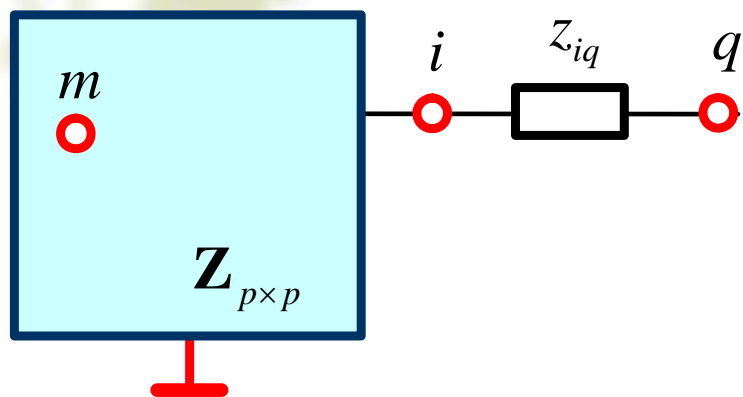
$$\dot{V}_m = Z_{mq} \dot{I}_q = Z_{mi} \dot{I}_q$$

$$\dot{V}_q = \dot{V}_i + z_{iq} \dot{I}_q = (Z_{ii} + z_{iq}) \dot{I}_q = Z_{qq} \dot{I}_q$$

$$Z_{mq} = Z_{mi}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加树枝



Z阵增加一行一列

Z阵原有元素不变

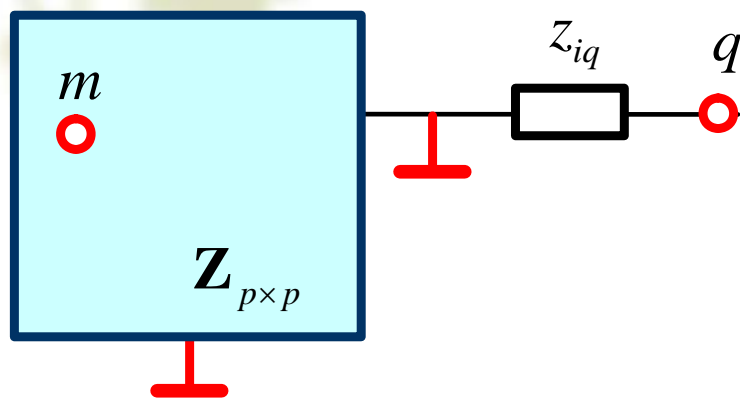
$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

$$Z_{mq} = Z_{mq} = Z_{mi}, (m = 1, 2, \dots, p)$$

$$Z_{qq} = Z_{ii} + z_{iq}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加接地树枝



Z阵增加一行一列

Z阵原有元素不变

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

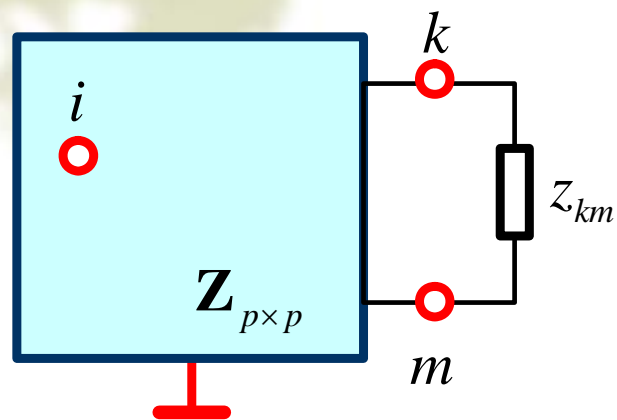
$$Z_{mq} = Z_{qm} = 0, (m = 1, 2, \dots, p)$$

$$Z_{qq} = Z_{iq}$$



## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



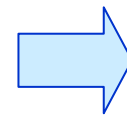
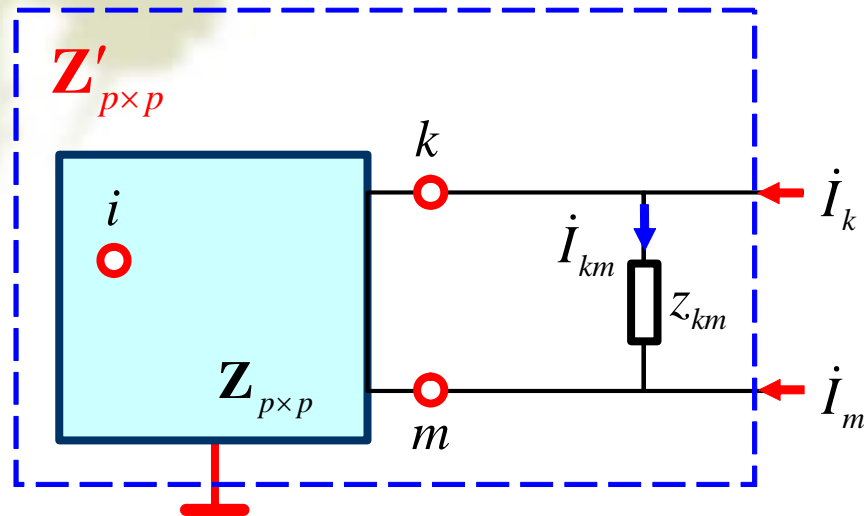
$$\mathbf{Z}_{p \times p} \rightarrow \mathbf{Z}'_{p \times p}$$

$$\begin{bmatrix} Z'_{11} & \cdots & Z'_{1m} & \cdots & Z'_{1k} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{m1} & \cdots & Z'_{mm} & \cdots & Z'_{mk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{k1} & \cdots & Z'_{km} & \cdots & Z'_{kk} & \cdots \\ \hline \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \end{bmatrix}$$

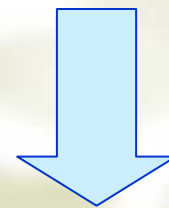
$Z_{km}$  支路会引起原网络电压电流分布的变化

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



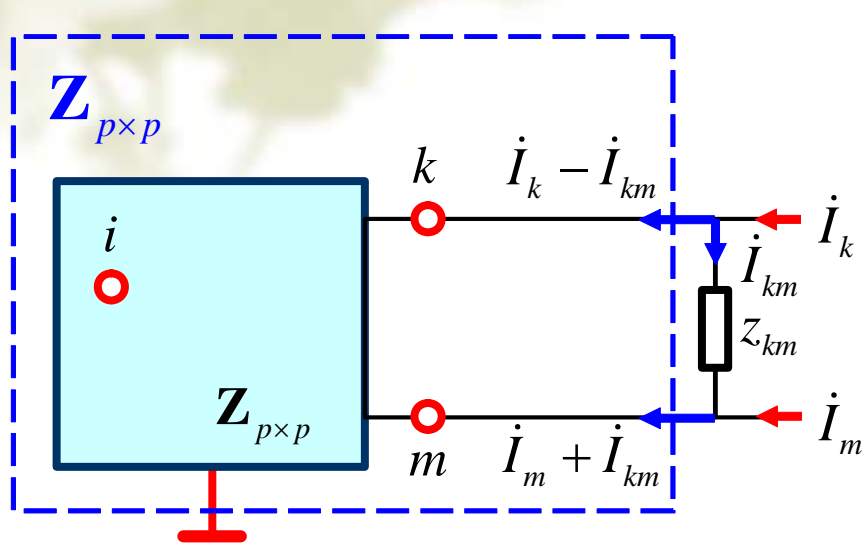
$$\dot{\mathbf{V}} = \mathbf{Z}' \dot{\mathbf{I}}$$



$$\dot{V}_i = \sum_{j=1}^p Z'_{ij} \dot{I}_j = Z'_{i1} \dot{I}_1 + \cdots + Z'_{ik} \dot{I}_k + \cdots + Z'_{im} \dot{I}_m + \cdots + Z'_{ip} \dot{I}_p$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



$$\dot{\mathbf{V}} = \mathbf{Z}\dot{\mathbf{I}}'$$

$$\dot{I}'_k = \dot{I}_k - \dot{I}_{km}$$

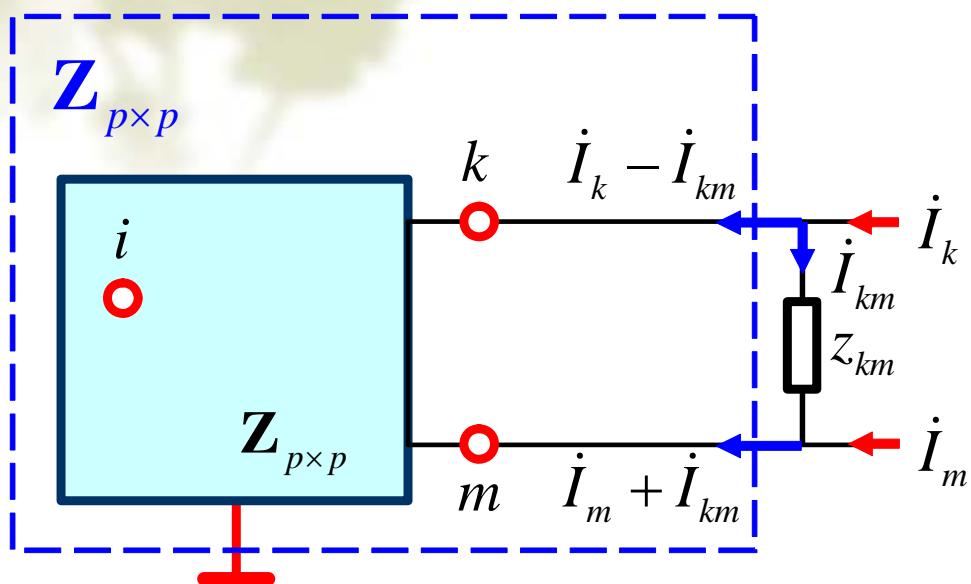
$$\dot{I}'_m = \dot{I}_m + \dot{I}_{km}$$

$$\dot{I}'_j = \dot{I}_j, j \neq k, m$$

$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}'_j = Z_{i1} \dot{I}_1 + \cdots + Z_{ik} (\dot{I}_k - \dot{I}_{km}) + \cdots + Z_{im} (\dot{I}_m + \dot{I}_{km}) + \cdots + Z_{ip} \dot{I}_p$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



$$\dot{V}_k = \sum_{j=1}^p Z_{kj} \dot{I}_j - (Z_{kk} - Z_{km}) \dot{I}_{km}$$

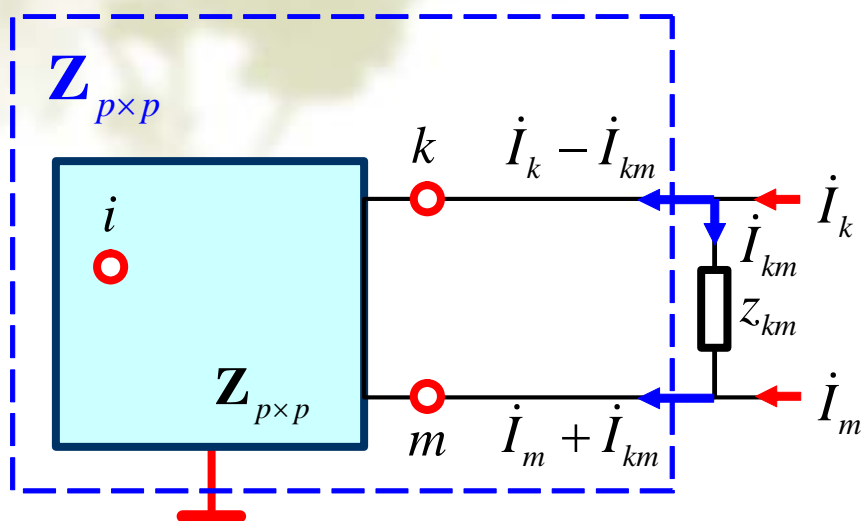
$$\dot{V}_m = \sum_{j=1}^p Z_{mj} \dot{I}_j - (Z_{mk} - Z_{mm}) \dot{I}_{km}$$

$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}_j - (Z_{ik} - Z_{im}) \dot{I}_{km}$$

$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}'_j = Z_{i1} \dot{I}_1 + \cdots + Z_{ik} (\dot{I}_k - \dot{I}_{km}) + \cdots + Z_{im} (\dot{I}_m + \dot{I}_{km}) + \cdots + Z_{ip} \dot{I}_p$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



$$\dot{V}_k = \sum_{j=1}^p Z_{kj} \dot{I}_j - (Z_{kk} - Z_{km}) \dot{I}_{km}$$

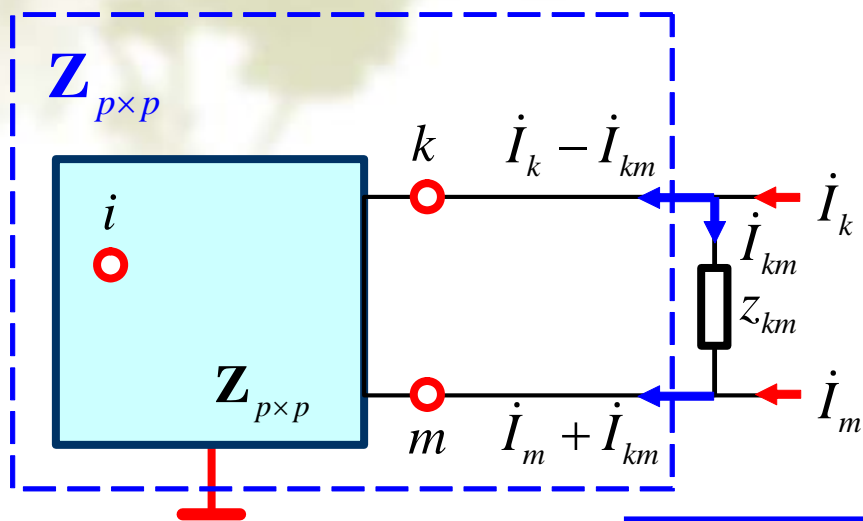
$$\dot{V}_m = \sum_{j=1}^p Z_{mj} \dot{I}_j - (Z_{mk} - Z_{mm}) \dot{I}_{km}$$

$$\dot{V}_k - \dot{V}_m = \sum_{j=1}^p (Z_{kj} - Z_{mj}) \dot{I}_j - (Z_{kk} - Z_{km} - Z_{mk} + Z_{mm}) \dot{I}_{km} = z_{km} \dot{I}_{km}$$

$$\dot{I}_{km} = \frac{1}{(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km})} \sum_{j=1}^p (Z_{kj} - Z_{mj}) \dot{I}_j$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



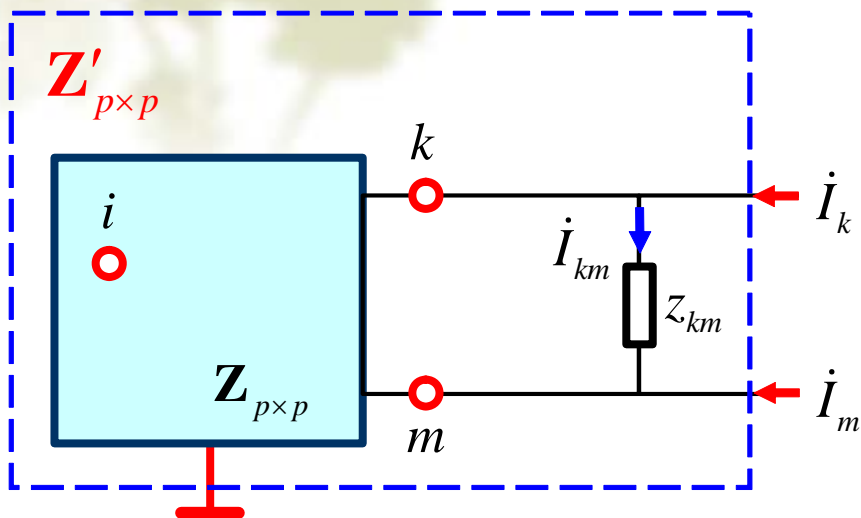
$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}_j - (Z_{ik} - Z_{im}) \dot{I}_{km}$$

$$\dot{I}_{km} = \frac{1}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} \sum_{j=1}^p (Z_{kj} - Z_{mj}) \dot{I}_j$$

$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}_j - \frac{(Z_{ik} - Z_{im})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} \sum_{j=1}^p (Z_{kj} - Z_{mj}) \dot{I}_j$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



$$Z'_{ij} = Z_{ij} - \frac{(Z_{ik} - Z_{im})(Z_{kj} - Z_{mj})}{(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km})} \quad (i, j = 1, 2, \dots, p)$$

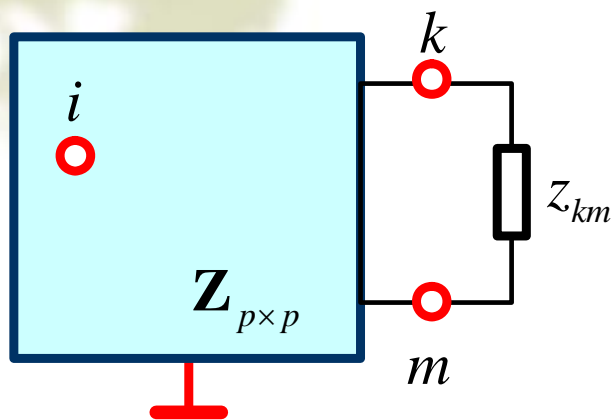
$$\dot{V}_i = \sum_{j=1}^p Z_{ij} \dot{I}_j - \frac{(Z_{ik} - Z_{im})}{(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km})} \sum_{j=1}^p (Z_{kj} - Z_{mj}) \dot{I}_j$$

$$\dot{V}_i = \sum_{j=1}^p Z'_{ij} \dot{I}_j$$

$$\dot{V}_i = \sum_{j=1}^p \left[ Z_{ij} - \frac{(Z_{ik} - Z_{im})(Z_{kj} - Z_{mj})}{(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km})} \right] \dot{I}_j$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加连支



$$\mathbf{Z}_{p \times p} \rightarrow \mathbf{Z}'_{p \times p}$$

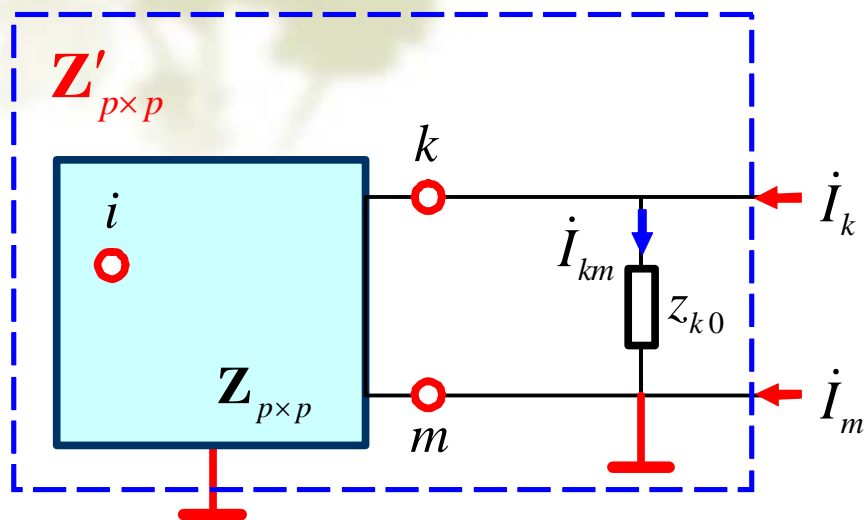
$$\begin{bmatrix} Z'_{11} & \cdots & Z'_{1m} & \cdots & Z'_{1k} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{m1} & \cdots & Z'_{mm} & \cdots & Z'_{mk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{k1} & \cdots & Z'_{km} & \cdots & Z'_{kk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \end{bmatrix}$$

$$Z'_{ij} = Z_{ij} - \frac{(Z_{ik} - Z_{im})(Z_{kj} - Z_{mj})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} \quad (i, j = 1, 2, \dots, p)$$



## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加接地连支



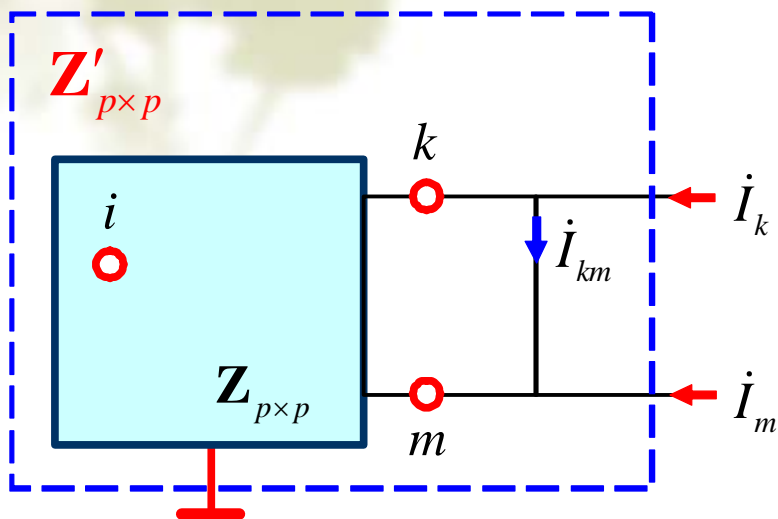
$$Z'_{ij} = Z_{ij} - \frac{(Z_{ik} - Z_{im})(Z_{kj} - Z_{mj})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{k0})} \quad (i, j = 1, 2, \dots, p)$$

$$Z_{im} = Z_{mj} = Z_{km} = Z_{mm} = 0$$

$$Z'_{ij} = Z_{ij} - \frac{Z_{ik}Z_{kj}}{Z_{kk} + Z_{k0}}, \quad (i, j = 1, 2, \dots, p)$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加零阻抗连支



$$Z'_{ij} = Z_{ij} - \frac{(Z_{ik} - Z_{im})(Z_{kj} - Z_{mj})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})}$$

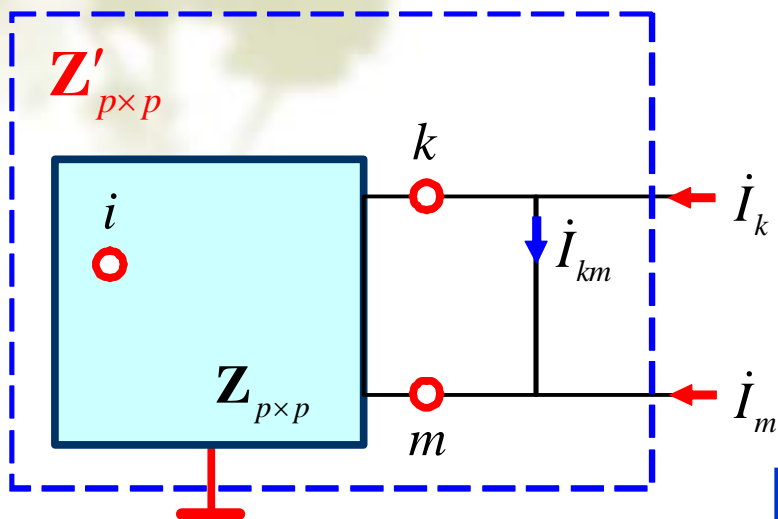
$$Z'_{ik} = Z_{ik} - \frac{(Z_{ik} - Z_{im})(Z_{kk} - Z_{mk})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})}$$

$$Z'_{im} = Z_{im} - \frac{(Z_{ik} - Z_{im})(Z_{km} - Z_{mm})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})}$$

$$Z'_{ik} - Z'_{im} = Z_{ik} - Z_{im} - \frac{(Z_{ik} - Z_{im})(Z_{kk} - Z_{mk})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} + \frac{(Z_{ik} - Z_{im})(Z_{km} - Z_{mm})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加零阻抗连支



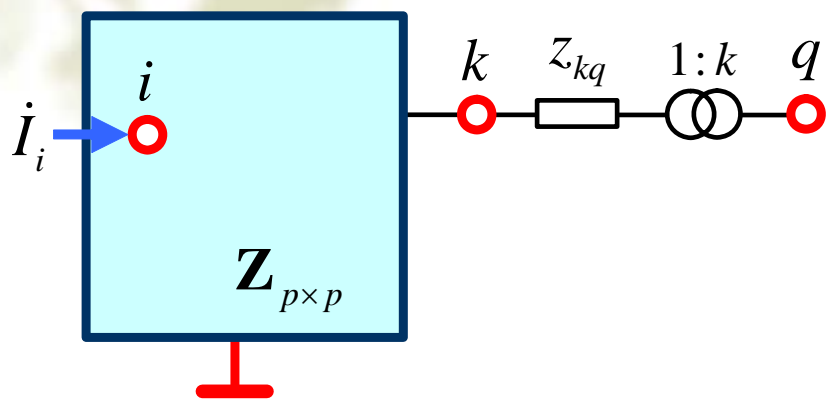
$Z_{km} = 0$ , 相当于  $k$ 、 $m$  节点合并, 则有  $Z'_{ik} - Z'_{im} = 0$ , 即第  $k$  列和第  $m$  列元素完全相等

$$Z'_{ik} - Z'_{im} = (Z_{ik} - Z_{im}) \left[ 1 - \frac{(Z_{kk} - Z_{mk}) - (Z_{km} - Z_{mm})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} \right]$$

$$Z'_{ik} - Z'_{im} = Z_{ik} - Z_{im} - \frac{(Z_{ik} - Z_{im})(Z_{kk} - Z_{mk})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})} + \frac{(Z_{ik} - Z_{im})(Z_{km} - Z_{mm})}{(Z_{kk} + Z_{mm} - 2Z_{km} + Z_{km})}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加变压器树支



$i$ 节点单独注入电流 $I_i$

$$\dot{V}_k = Z_{ki} \dot{I}_i$$

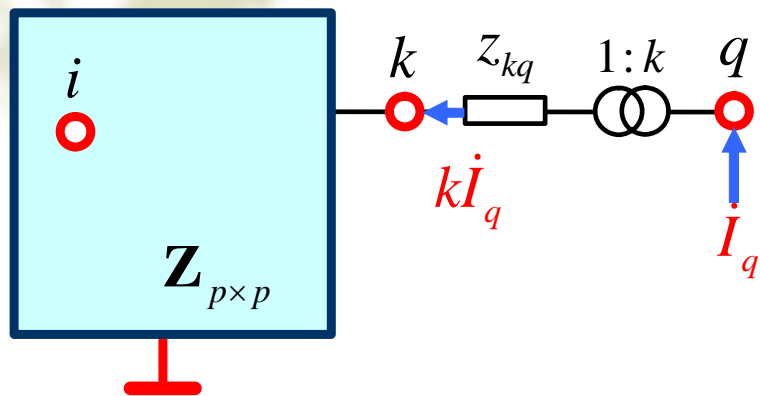
$$\dot{V}_q = Z_{qi} \dot{I}_i = k \dot{V}_k = k Z_{ki} \dot{I}_i$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

$$\Rightarrow Z_{qi} = k Z_{ki}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加变压器树支



$Z_{11}$	$\cdots$	$Z_{1i}$	$\cdots$	$Z_{1p}$	$Z_{1q}$
$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$Z_{k1}$	$\cdots$	$Z_{kk}$	$\cdots$	$Z_{kp}$	$Z_{kq}$
$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$Z_{p1}$	$\cdots$	$Z_{pk}$	$\cdots$	$Z_{pp}$	$Z_{pq}$
$Z_{q1}$	$\cdots$	$Z_{qk}$	$\cdots$	$Z_{qp}$	$Z_{qq}$

$q$ 节点单独注入电流  $I_q$

$$\dot{V}_i = Z_{iq} I_q = kZ_{ik} I_q$$

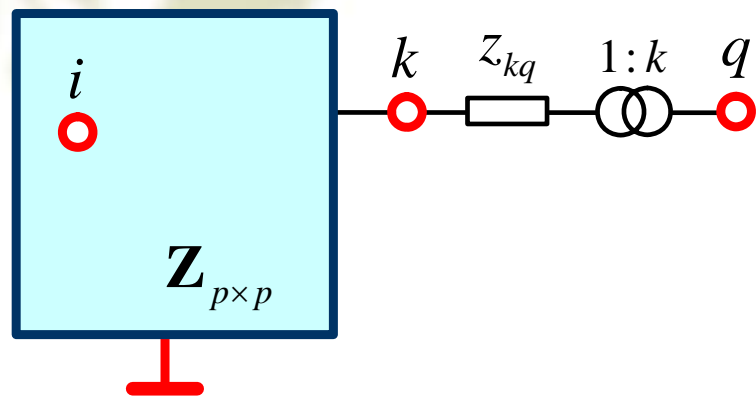


$$Z_{iq} = kZ_{ik}$$

$$\dot{V}_q = k(\dot{V}_k + z_{kq} kI_q) = k^2(Z_{kk} + z_{kq}) I_q = Z_{qq} I_q$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加变压器树支



Z阵增加一行一列

Z阵原有元素不变

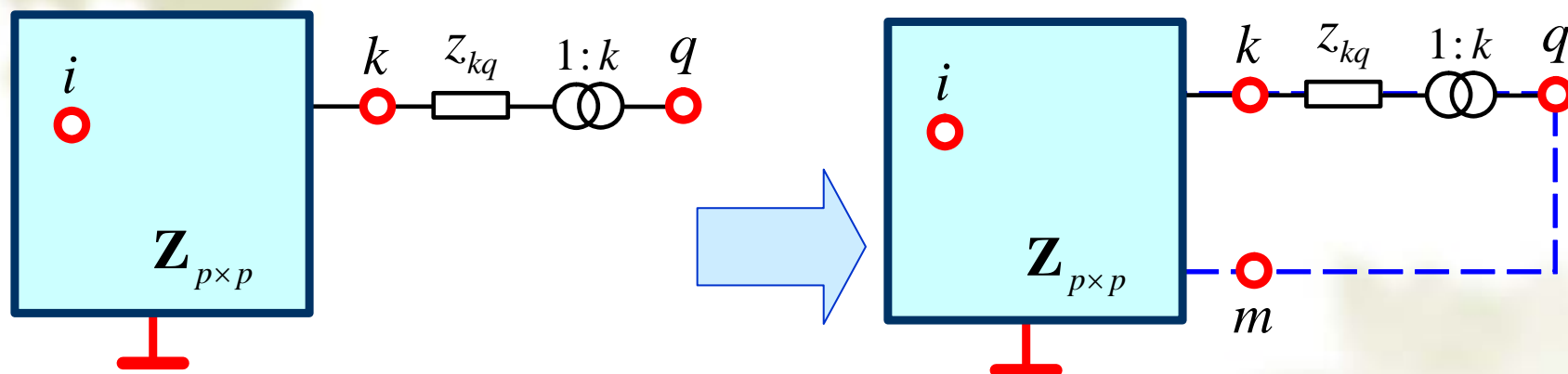
$$Z_{iq} = Z_{qi} = kZ_{ki}, (i = 1, 2, \dots, p)$$

$$Z_{qq} = k^2 (Z_{kk} + z_{kq})$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加变压器连支



$$\mathbf{Z}_{p \times p} \rightarrow \mathbf{Z}_{(p+1) \times (p+1)}$$

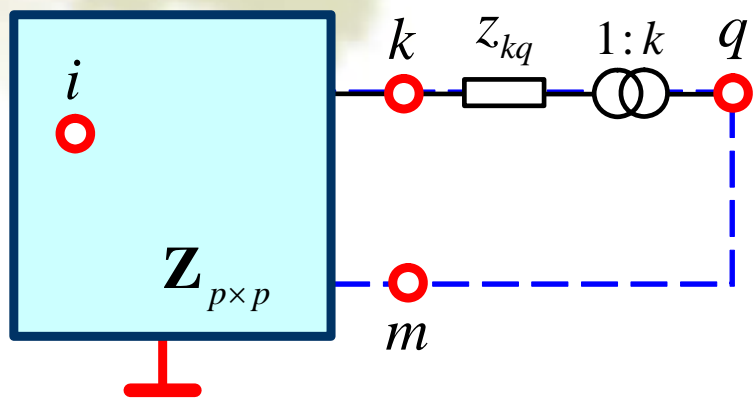
$$\mathbf{Z}'_{p \times p} \leftarrow \mathbf{Z}'_{(p+1) \times (p+1)}$$

$$\begin{aligned} Z_{iq} &= Z_{qi} = kZ_{ik} \\ Z_{qq} &= k^2 (Z_{kk} + z_{kq}) \end{aligned}$$

$$Z'_{ij} = Z_{ij} - \frac{(Z_{iq} - Z_{im})(Z_{qj} - Z_{mj})}{(Z_{qq} + Z_{mm} - 2Z_{qm})}$$

## 4-3 节点阻抗矩阵

### 3. 支路追加法生成Z阵-追加变压器连支



$$Z'_{ij} = Z_{ij} - \frac{(Z_{iq} - Z_{im})(Z_{qj} - Z_{mj})}{(Z_{qq} + Z_{mm} - 2Z_{qm})}$$

$$Z'_{p \times p}$$

$$Z_{iq} = Z_{qi} = kZ_{ik}, Z_{qq} = k^2(Z_{kk} + z_{kq})$$

$$Z'_{ij} = Z_{ij} - \frac{(kZ_{ik} - Z_{im})(kZ_{kj} - Z_{mj})}{k^2(Z_{kk} + z_{kq}) + Z_{mm} - 2kZ_{mk}}, (i, j = 1, 2, \dots, p)$$



## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $\mathbf{YZ}=\mathbf{I}$ 计算 $\mathbf{Z}$ 阵

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\mathbf{Y} [\mathbf{Z}_1 \ \mathbf{Z}_2 \ \cdots \ \mathbf{Z}_n] = [\mathbf{e}_1 \ \mathbf{e}_2 \ \cdots \ \mathbf{e}_n]$$

$$\mathbf{Z}_j = [Z_{1j} \ Z_{2j} \ \cdots \ Z_{nj}]^T$$

$$\mathbf{Y}\mathbf{Z}_j = \mathbf{e}_j, (j = 1, 2, \cdots, n)$$

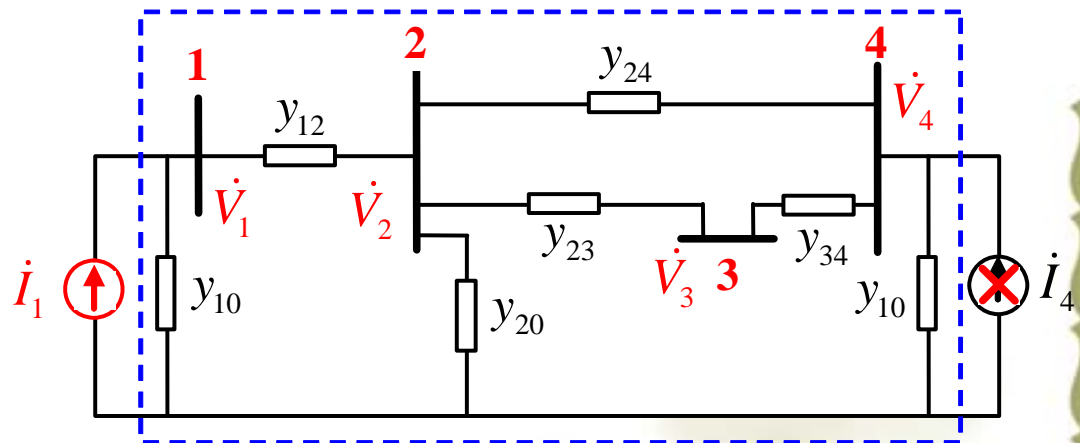
$$\mathbf{e}_j = [0 \ \cdots \ 1 \ \cdots \ 0]^T$$

第  $j$  列

## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $YZ=I$ 计算 $Z$ 阵

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$



$$YZ_j = e_j$$

$$Y\dot{V} = \dot{I}$$

$$\begin{aligned} \dot{I}_i &= 0, \\ \dot{I}_j &= 1, \\ &(i \neq j) \end{aligned}$$

$$\begin{aligned} \dot{V}_j &= Z_{jj} \dot{I}_j = Z_{jj} \\ \dot{V}_i &= Z_{ij} \dot{I}_j = Z_{ij} \end{aligned}$$

方程的物理意义

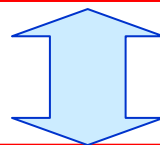
## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $YZ=I$ 计算 $Z$ 阵

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$YZ_j = e_j \quad LDUZ_j = e_j$$

$$Y = LDU$$



$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ l_{21} & 1 & & & \\ l_{31} & l_{32} & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} d_{11} & & & & \\ & d_{22} & & & \\ & & d_{33} & & \\ & & & \ddots & \\ & & & & d_{nn} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ & 1 & u_{23} & \cdots & u_{2n} \\ & & 1 & \cdots & u_{3n} \\ & & & \ddots & \vdots \\ & & & & 1 \end{bmatrix}$$

## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $\mathbf{YZ}=\mathbf{I}$ 计算 $\mathbf{Z}$ 阵

$$\mathbf{LDUZ}_j = \mathbf{e}_j$$

$$\mathbf{LF} = \mathbf{e}_j$$

$$f_i = \begin{cases} 0 & i < j \\ 1 & i = j \\ -\sum_{k=j}^{i-1} l_{ik} f_k, & i > j \end{cases}$$

$$\begin{bmatrix} 1 & & & & & & & & \\ l_{21} & 1 & & & & & & & \\ l_{31} & l_{32} & 1 & & & & & & \\ \vdots & \vdots & \vdots & \ddots & & & & & \\ l_{j1} & l_{j2} & l_{j3} & \cdots & 1 & & & & \\ \vdots & \vdots & \vdots & & \vdots & \ddots & & & \\ l_{i1} & l_{i2} & l_{i3} & \cdots & l_{ij} & \cdots & 1 & & \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \ddots & & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nj} & \cdots & l_{ni} & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_j \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $YZ=I$ 计算 $Z$ 阵

$$LDUZ_j = e_j$$

$$DUZ_j = F$$

$$DH = F$$

$$h_i = \begin{cases} 0 & i < j \\ f_i/d_{ii} & i \geq j \end{cases}$$

$$\begin{bmatrix} d_{11} & & & & & \\ & d_{22} & & & & \\ & & d_{33} & & & \\ & & & \ddots & & \\ & & & & d_{jj} & \\ & & & & & \ddots & \\ & & & & & & d_{ii} & \\ & & & & & & & \ddots & \\ & & & & & & & & d_{nn} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_j \\ \vdots \\ h_i \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_j \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix}$$

## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $\mathbf{YZ}=\mathbf{I}$ 计算 $\mathbf{Z}$ 阵

$$\mathbf{LDUZ}_j = \mathbf{e}_j$$

$$\mathbf{LF} = \mathbf{e}_j$$

$$\mathbf{DUZ}_j = \mathbf{F}$$

$$\mathbf{DH} = \mathbf{F}$$

$$\mathbf{UZ}_j = \mathbf{H}$$

$$\begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1j} & \cdots & u_{1i} & \cdots & u_{1n} \\ & 1 & u_{23} & \cdots & u_{2j} & \cdots & u_{2i} & \cdots & u_{2n} \\ & & 1 & \cdots & u_{3j} & \cdots & u_{3i} & \cdots & u_{3n} \\ & & & \ddots & \vdots & & \vdots & & \vdots \\ & & & & 1 & & u_{ji} & \cdots & u_{jn} \\ & & & & & \ddots & \vdots & & \vdots \\ & & & & & & 1 & \cdots & u_{in} \\ & & & & & & & \ddots & \vdots \\ & & & & & & & & 1 \end{bmatrix} \begin{bmatrix} Z_{1j} \\ Z_{2j} \\ Z_{3j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_j \\ \vdots \\ h_i \\ \vdots \\ h_n \end{bmatrix}$$

$$Z_{ij} = h_i - \sum_{k=i+1}^n u_{ik} Z_{kj}, \quad i = n, n-1, \dots, 1$$

## 4-3 节点阻抗矩阵

### 4. 由线性代数方程 $\mathbf{YZ}=\mathbf{I}$ 计算 $\mathbf{Z}$ 阵-因子表

$$\mathbf{L}^T = \mathbf{U}$$

$$l_{ki} = u_{ik}$$

$$\begin{bmatrix} d_{11}^{-1} & u_{12} & u_{13} & \cdots & u_{1j} & \cdots & u_{1i} & \cdots & u_{1n} \\ & d_{22}^{-1} & u_{23} & \cdots & u_{2j} & \cdots & u_{2i} & \cdots & u_{2n} \\ & & d_{33}^{-1} & \cdots & u_{3j} & \cdots & u_{3i} & \cdots & u_{3n} \\ & & & \ddots & \vdots & & \vdots & & \vdots \\ & & & & d_{jj}^{-1} & & u_{ji} & \cdots & u_{jn} \\ & & & & & \ddots & \vdots & & \vdots \\ & & & & & & d_{ii}^{-1} & \cdots & u_{in} \\ & & & & & & & \ddots & \vdots \\ & & & & & & & & d_{nn}^{-1} \end{bmatrix}$$



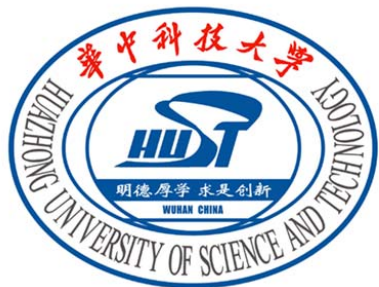
华中科技大学

Huazhong University of  
Science and Technology

## 本章小结

- ❖ **Y** 阵元素的物理意义；**Y** 阵的特点、形成和修改；
- ❖ **Z** 阵元素的物理意义，根据**Z** 阵元素的物理意义形成**Z** 阵的方法；
- ❖ 利用线性代数方程  $\mathbf{YZ}_j = \mathbf{e}_j$  计算**Z** 阵的某一系列元素；

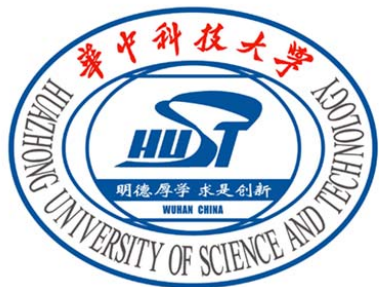




华中科技大学  
Huazhong University of  
Science and Technology

# 习题

**Ex 4-1, 4-2, 4-4**



华中科技大学  
Huazhong University of  
Science and Technology



**To Be Continued**